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J E WIESELTHIER ET AL. 30 AUG 84 NRL-MR-5424

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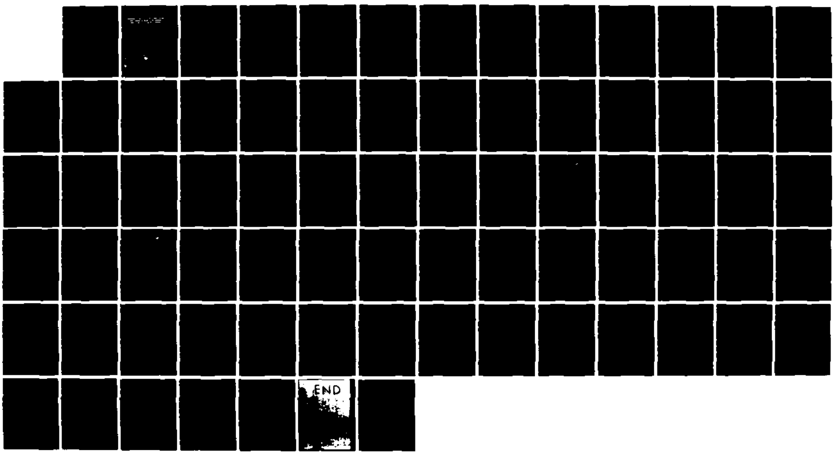

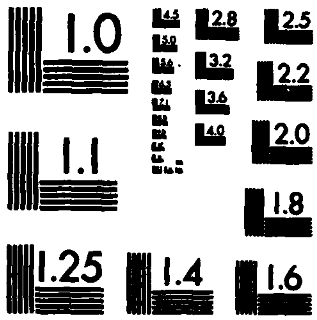

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In this report we derive the probability of correct packet reception and the resulti:g channel throughput achievable in an asynchronous slow-frequency-hopped multiple user channel. Reed-Solomon coding is used to correct errors caused by other-user interference in an otherwise noiseless channel. We analyze and evaluate an M-ary FSK signaling scheme, which permits the discrimination against interfering signals that are present for a sufficiently small fraction of the hop duration, and results in substantial increases in channel throughput over previous models.

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CONTENTS

THROUGHPUT INCREASE IN FREQUENCY HOPPED MULTIPLE ACCESS CHANNELS BY MEANS OF DISCRIMINATION AGAINST PARTIALLY OVERLAPPING INTERFERENCE

1. INTRODUCTION

The use of spread spectrum signaling permits the sharing of a wideband channel by means of Code Division Multiple Access (CDMA) techniques. The number of users that can share such a wideband channel simultaneously and the resulting performance depend on the modulation/coding scheme, channel characteristics, and receiver implementation. In this paper we consider frequency hopping (FH) spread spectrum multiple access systems in which Reed-Solomon coding is used to correct burst errors caused by other-user interference in a packet-switched environment. We extend the model of Pursley and Hajek [1-4], and evaluate a signaling scheme that, for the case of a noiseless channel, provides considerable improvement in channel throughput.

We begin this paper with a brief description of the system model. A slow FH system is considered, i.e., one in which one symbol corresponding to one or more bits is transmitted per hop. Network timing is asynchronous, although each receiver is assumed to be perfectly synchronized to the desired signal. We address only the case of a fixed number of users that continuously transmit over the channel, although this model can be extended to include the case of bursty users. Randomness in the system

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therefore arises not because of changing user demands, but rather because of the pseudorandom nature and asynchronous timing of the hopping patterns.

In [1-4] two models are used to consider the effect of other-user interference on individual hops. In the first, all hops that are affected by other-user interference are assumed to be received with undetectable errors. In the second model it is assumed that side information is available, i.e., knowledge of which of the received hops have been corrupted by other-user interference; such hops can be erased, thereby resulting in considerable performance improvement. In this paper we propose a third model (originally introduced in [5,6,7]) for other-user interference in which hops can be correctly received, despite the partial overlap by other users' signals, provided that the overlap is a sufficiently small fraction of the hop duration. This ability to discriminate against interfering signals results in further substantial performance improvement. The interference model is based on the use of M-ary frequency shift keying (FSK) signaling (multiple parallel binary FSK signaling is also considered), and the ability of the receiver to discriminate against interfering signals that are present for a sufficiently small fraction of the hop duration. We discuss how such a scheme might be implemented, and we calculate the performance improvement.

2. THE BASIC MODEL

We consider a wideband frequency hopping (FH) channel that consists of q orthogonal narrowband frequency bins. We assume the use of noncoherent M-ary FSK; each frequency bin thus consists of M orthogonal tone positions. Each user of the channel transmits one fixed length string of symbols, called a packet, in each time slot. Each symbol, consisting of a single tone representing $\log_2 M$ bits, is transmitted in one hop. Figure 1 illustrates this signaling scheme. The frequency hopping patterns, which must be known by both the transmitter and receiver, are assumed to be generated by a pseudorandom first order Markov process such that each frequency bin is different from the previous one, but equally likely to be any one of the q-1 other frequencies.* We assume that perfect synchronization is maintained between transmitter and receiver. Furthermore, we assume a noiseless channel model in which the only interference is that which is caused by other users.

In FH-CDMA systems the code corresponds to the FH pattern. CDMA operation is usually asynchronous at the hop level, and therefore it is possible for two or more users (even when they use orthogonal hopping patterns) to transmit simultaneously in

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* Although other types of pseudorandom hopping patterns can also be accommodated in our model the hopping scheme considered here is more easily analyzed when partial overlaps can be discriminated against, as is discussed later.

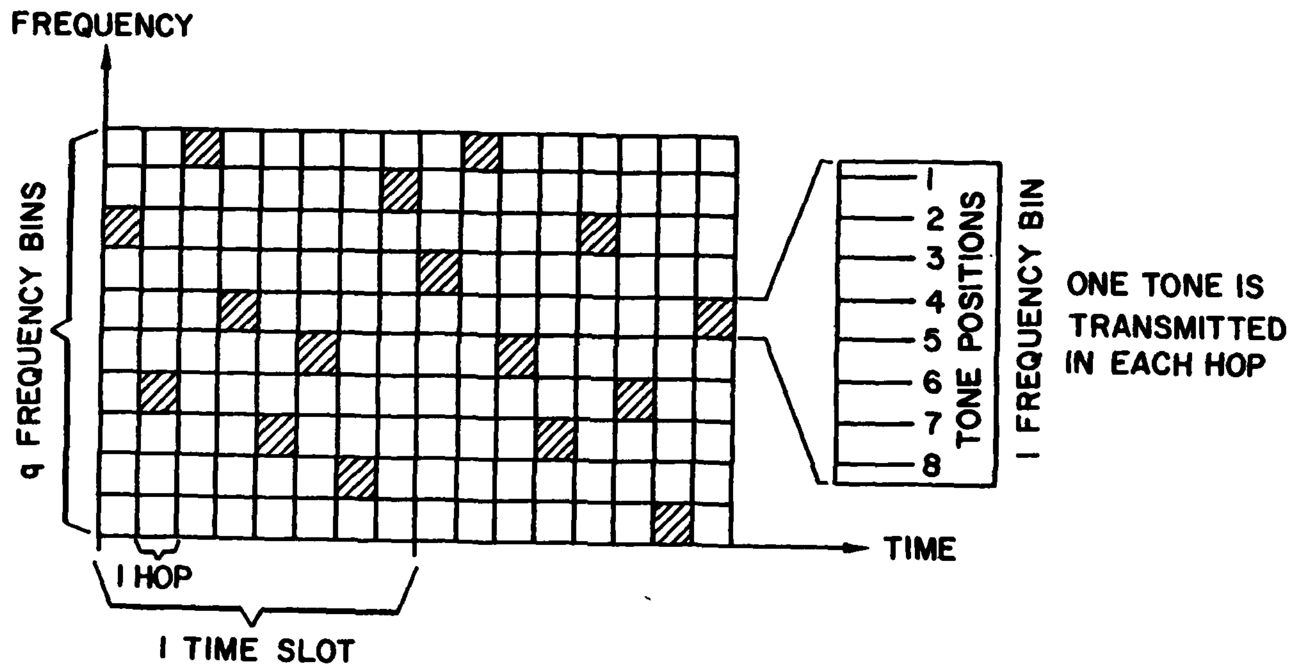

Fig. 1 Frequency hopping channel with one M-ary FSK symbol transmitted per hop and one packet (codeword) transmitted per time slot (example shown for $M = 8$).

the same frequency bin, resulting in loss of data. Such collisions (of fractions of packets in this case as compared with whole packets in the usual case of "time-domain" multiple access schemes), are known as "hits." As a further consequence of the asynchronous nature of the FH process the occurrence of hits generally results in partial, rather than total, overlap among tones of different users; it is assumed that the degree of overlap experienced by any pair of hops that suffer a hit is uniformly distributed over the interval from 0 to the total hop duration value. It is usually assumed that any degree of hop overlap results in the loss of the information carried in that hop. In this paper we make a departure from this assumption.

We address only the case of a fixed number of users that continuously transmit over the channel, although the model presented in this paper can be extended to include the case of bursty users as well, as was done by Hajek [2]. Each user transmits one packet in each time slot. Each packet consists of a Reed-Solomon (RS) codeword of $n = M-1$ M-ary symbols, one of which is transmitted per hop. A RS-(n,v) code is capable of correcting $t = (n-v)/2$ symbol errors in any codeword (packet). Furthermore, as was demonstrated in [8], the probability that a RS codeword error occurs without being detected is less than $1/t!$ (less than 2×10^{-5} for the RS-(31,15) code and 10^{-89} for the RS-(255,127) code), and is thus negligible in many applications.

Considerable improvement can be obtained in a system that is capable of detecting which of the received symbols have been affected by frequency hits, and of erasing the corresponding symbols. In the general case in which both errors and erasures occur, a correct codeword decision can be made as long as the number of symbol erasures plus twice the number of symbol errors is not greater than 2t. The detection of hits would be straightforward in applications involving noiseless channels and M-ary FSK signaling. The presence of energy in more than one tone position at the same time (a situation impossible for a valid signal) would indicate the occurrence of a hit, and thus all hits could be detected and erased.*

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* If another user's signal destructively interferes with the desired signal so as to essentially cancel it, and if a third signal is present then the latter may be perceived as the desired signal. For such an undetected symbol error to occur the interfering signal would have to be in the same tone position as the desired signal, would have to overlap for most of the hop duration, be close to 180° out of phase with the desired signal, and be of almost identical amplitude. We neglect such relatively infrequent events in our analysis.

To calculate the performance of a FH multiple user system we need to compute certain quantities that lead to the evaluation of the channel throughput. By throughput we mean the expected number of successful packets that can be delivered per time slot, where a time slot is equal to a packet duration (and is therefore different for the different RS codes and packet sizes that are considered).*

Consider a particular fixed user who transmits a packet in a given slot. Let there be k additional active users during that slot. We define,

$\Pr[E|k]$ = Pr[the packet of the fixed user is not correctly received| given that there are k <u>other</u> users on the channel].

Then the conditional throughput, given a total of (k+1) users, can be evaluated as,

$$S_{k+1} = (k+1)(1 - \Pr[E|k]), \tag{1}$$

or, if we want to normalize with respect to the frequency bin bandwidth,

$$s_{k+1} = (1/q)S_{k+1} \tag{2}$$

which is the expected number of successful packets per time slot and frequency bin. Recall that we have assumed that each of the k+1 users transmits a packet in every time slot, and so the only

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* This definition of throughput implies that packet errors can be detected, and that all packets that are incorrectly received are subsequently retransmitted (an acknowledgment mechanism is of course needed to do so). We noted earlier that the use of RS coding does in fact permit the detection of virtually all codeword errors.

randomness that arises in this model is that which results from the pseudorandom nature of the FH patterns.* Also, we assume that the tone positions in each bin are orthogonally spaced; i.e., their spectral distance is the reciprocal of the symbol (hop) duration. If the tones are closer together it is more difficult to discriminate between adjacent tone positions. Therefore, the minimum duration of a tone (i.e., the hop duration or hop dwell time) that is consistent with the orthogonal tone spacing requirement is uniquely determined by the tone spacing. If the dwell time is indeed equal to the minimum possible value that ensures tone orthogonality we shall say that the dwell time is <u>matched</u> to the tone spacing.

Fundamental to the evaluation of $P[E|k]$ is the evaluation of the symbol error probability. We define,

p_k = Pr[a given symbol is not received correctly| given that there are k <u>other</u> users on the channel].

This quantity depends on the model that is assumed for other-user interference as well as on q. Let us first assume pessimistically that all frequency hits result in symbol errors. The symbol error probability for this pessimistic model, given that k <u>other</u> users are simultaneously transmitting over the channel and that timing is asynchronous was shown in [3] to be,

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* If the users are bursty we can average the conditional throughput described above with respect to the statistics of the packet generation and retransmission process to obtain the unconditional throughput. Such a calculation is generally complicated and goes beyond the scope of this paper. (See e.g., Hajek [2].)

$$p_k = 1 - (1 - 2/q)^k. \qquad (3)$$

The quantity $1/q$ represents the probability that a user chooses a specific bin for transmission during a given hop. Since there is no hop synchronization between users, partial overlap may occur either from the left or from the right, as illustrated in Fig. 2. Thus, the probability that a specific one of the other k users will interfere with the symbol of the user of interest is $2/q$. Recall that we do not allow use of the same bin in two consecutive hops; we can easily relax this assumption by replacing $2/q$ with $2/q - 1/q^2$. We can also accommodate other hopping patterns that yield different expressions for p_k.

The symbol error process for any user is a sequence of independent Bernoulli trials, because of the pseudorandom nature of the hopping patterns (and therefore of the interference process). Since the Reed-Solomon codes under consideration can tolerate t symbol errors in any n-symbol codeword, the packet error probability, given k other users, is:

$$Pr(E|k) = \sum_{i=t+1}^{n} \binom{n}{i} p_k^i (1-p_k)^{n-i}. \qquad (4)$$

Since virtually all packet errors are detectable, packets that are received incorrectly can be retransmitted subsequently, provided of course that an acknowledgment mechanism can be implemented. Thus we can interpret the packet error probability as a <u>packet erasure</u> probability.

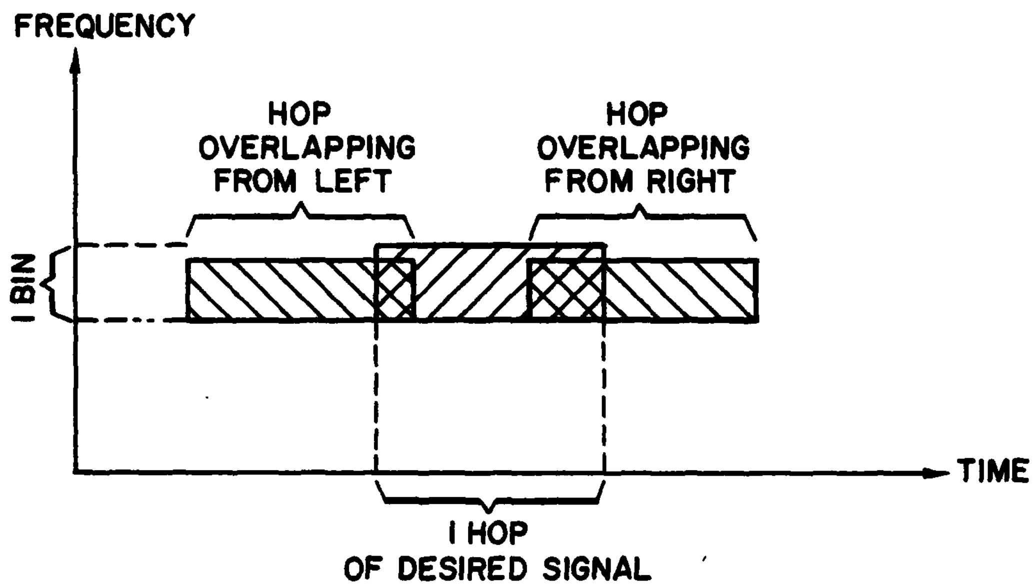

Fig. 2 A frequency hit in which two other users transmit in same frequency bin as desired signal, one overlapping from the left and one from the right.*

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* Note that signals are depicted with different amplitudes only to enhance the clarity of the figure. The amplitudes of the overlapping signals can be greater than that of the desired signal without affecting the analysis.

If all hits are detected and the corresponding symbols erased we simply change the lower limit of the summation in eq. (4) to 2t+1. The quantity p_k, which now represents symbol erasure probability, is still given by eq. (3).

We now turn our attention to the modified model that permits within-hit discrimination against other user interference.

3. THE MODIFIED MODEL IN WHICH PARTIAL OVERLAPS CAN BE TOLERATED

We previously assumed that any tone occupancy within the same bin as the desired signal by other users, even if only partially overlapping in time with the symbol of the user of interest, caused interference (thus necessitating an erasure, or perhaps resulting in a symbol error, depending on which interference model is considered). We again assume a noiseless channel; thus, there are no symbol errors in the absence of other-user interference. Also, we again assume that all hits can be detected. In addition, we now assume that a symbol erasure will be necessitated if and only if in any one (or more) of the M tone positions of the frequency bin the amount of time overlap between the symbol (tone) of interest and those of other users exceeds a fraction ρ of the hop duration time.* Otherwise, the symbol is received correctly.

Thus we must examine each of the M tone positions of the frequency bin to determine whether any of them experience interference for more than a fraction ρ of the hop duration. Note that this interference may arise from one or more other users' signals in the same tone position whose combined overlap at the same or at opposite ends of the hop lasts for a fraction of a hop greater than ρ.

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*This assumption requires discussion and interpretation. This is done in Section 4.

To make the definition of overlap clear we have illustrated in Fig. 3 the case of a number of overlapping signals, all of which are in the same tone position (possibly, but not necessarily, the same as that of the desired signal). There are α signals overlapping at the left edge of the hop and β signals overlapping at the right edge. The total overlap in the tone position we are considering is defined to be

$$\tau = \min(1, \tau_L + \tau_R) \tag{5}$$

where,

τ_L = maximum overlap of signals from left in this tone position,

τ_R = maximum overlap of signals from right in this tone position,

and τ_L and τ_R range from 0 (if no overlap) to 1 (if total overlap). Note that we do not combine overlaps in different tone positions, but rather treat each tone position separately.

A symbol erasure is thus necessary if and only if

$$\tau > \rho \tag{6}$$

in one or more of the M tone positions. An equivalent way of viewing the overlap model is that interference in a tone position will necessitate an erasure <u>if and only if the clear interval of the hop (i.e., the part containing no other users) is less than $1 - \rho$</u>.

Note that we have not distinguished the case in which the overlapping signals are in the same tone position as the desired signal. In that case our model is pessimistic, because in many cases the energy in the overlapping signal simply adds to the

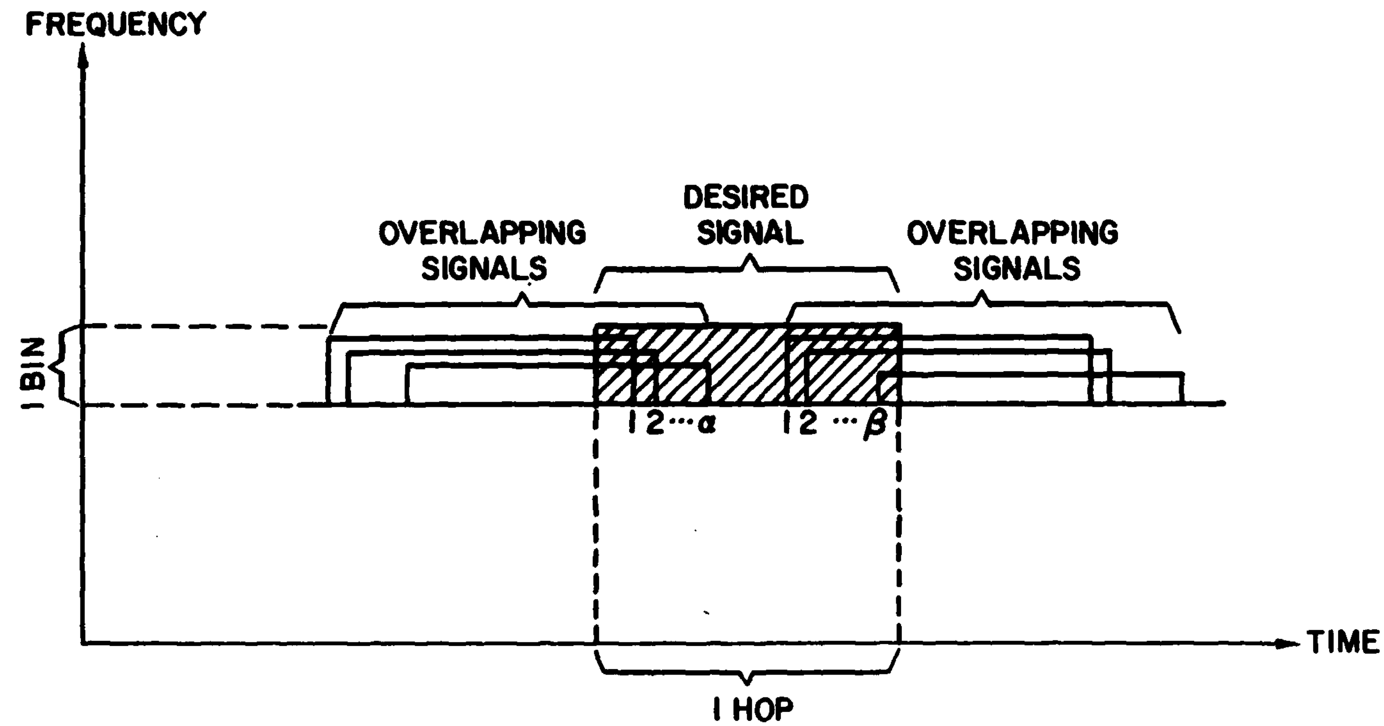

Fig. 3 The case of α signals overlapping from left and β signals overlapping from right in a single tone position.

energy in the desired signal. However, phase difference between the signals may produce amplitude cancellation, and it is thus safer to assume that a hit in the same tone as the desired signal results in interference.

The assumed ability to ignore interfering signals that are present for less than a certain fraction of the hop duration depends strongly on the fact that the M-ary FSK signal is constant throughout the hop duration. By contrast, in the case of binary FSK in which $\log_2 M$ bits are transmitted serially within one hop, if one or more bits are obscured by a hit there is no way to recover the lost information (unless additional coding is used within each hop), thus necessitating an erasure.

The evaluation of system performance proceeds in the same manner as presented earlier for the other interference models. To evaluate the probability of incorrect packet reception we again use eq. (4) with a lower summation limit of $i = 2t+1$. However, p_k, the symbol erasure probability given that a total of $k+1$ users transmit simultaneously, must be evaluated for the new interference model.

To evaluate p_k we again consider the case of k other users simultaneously using the channel along with the desired signal. We consider an arbitrary symbol of the desired signal, which corresponds to a specific frequency bin and a particular tone position therein. Suppose m out of the k other users have chosen the same frequency bin as the desired signal. Of course m is a random variable. We can express p_k conditioned on m as follows:

$$p_k = \sum_{m=1}^{k} P(e|m)Q(m|k) \tag{7}$$

where,

$P(e|m)$ = Pr(symbol erasure | m other users in same frequency bin as desired signal),

$Q(m|k)$ = Pr(m other users in same frequency bin as desired signal | k other users in the channel).

We sometimes refer to m as the number of hits during the hop.

Since the users are assumed to choose bins independently and with uniform distribution we have

$$Q(m|k) = \binom{k}{m} \left(\frac{2}{q}\right)^{m} \left(1 - \frac{2}{q}\right)^{k-m}. \tag{8}$$

Again the lack of symbol synchronism accounts for the factor 2 in eq. (8), since either one of two consecutive hops of another user may occupy the bin of interest.*

Thus it remains to evaluate the quantity $P(e|m)$. This calculation is done in a straightforward manner and utilizes combinatorial occupancy arguments. These tend to become somewhat complicated and thus we present them in Appendix A.

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*Our assumed hopping patterns exclude the possibility that two consecutive hops of the same user can be in the same tone position. If we do not exclude this possibility, then whenever two consecutive hops of an interfering signal are in the same frequency bin as the desired signal an erasure would be necessitated with probability one. The analysis of this scheme would require separate consideration of this special case, and would thus become complicated without providing additional insight.

4. ON THE INTERPRETATION OF THE MAXIMUM TOLERABLE OVERLAP PARAMETER

At this point we would like to discuss further the significance of the maximum tolerable overlap parameter and factors related to the implementation of systems that can tolerate some degree of hop overlap. To put things in perspective we can compare performance with that of a "baseline system" in which the hop duration is "matched" to (i.e., is the reciprocal of) the tone spacing, as discussed earlier, and in which all hits necessitate erasures, regardless of overlap duration.

The system we have been considering thus far in this paper is also one in which the hop duration is matched to the tone spacing, but partial overlaps less than ρ can be tolerated by virtue of the use of sophisticated receiver techniques.

For example, a possible implementation would consist of banks of matched filters, one for each tone. The outputs of these filters would be examined throughout the hop duration. If the time derivative of the output were zero the decision would be made that there is no signal at that tone frequency at that instant of time, independent of the total energy accumulated thus far. A signal would be declared present as long as the time derivative was sufficiently large.

The fact that partial overlaps can be tolerated suggests that we may be dwelling longer than we really have to in each frequency bin. In so doing we may be wasting energy (since only a fraction $1-\rho$ of the hop appears to be really needed) and transmitting data slower than we may be able to (by the same factor of $1-\rho$). We note however, that a shortening of the dwell time results in an increase in the orthogonal tone spacing, and therefore a decrease in the number of disjoint frequency bins (again by the factor of $1-\rho$) that are available within the same given fixed total bandwidth if we are to maintain a matched system. We are not concerned in this paper with the energy that may be "wasted" by dwelling too long. Our primary concern is the efficiency of the use of channel bandwidth, rather than of energy which we assume to be unconstrained.*

A Reduced Duty Cycle System.

Let us now consider a modification of the system studied in this paper. Suppose now that the system cannot tolerate partial overlap, but its dwell time is reduced to a fraction $(1-\rho)$ of the original dwell time, while the hopping rate remains the same; we would thus have a reduced duty cycle system, as shown in Figure 4. The expected number of other users that interfere with any hop of the desired signal is therefore reduced, but all hits,

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* Obviously, in general, energy savings must be traded off against spectral efficiency, a common design feature of most communication systems.

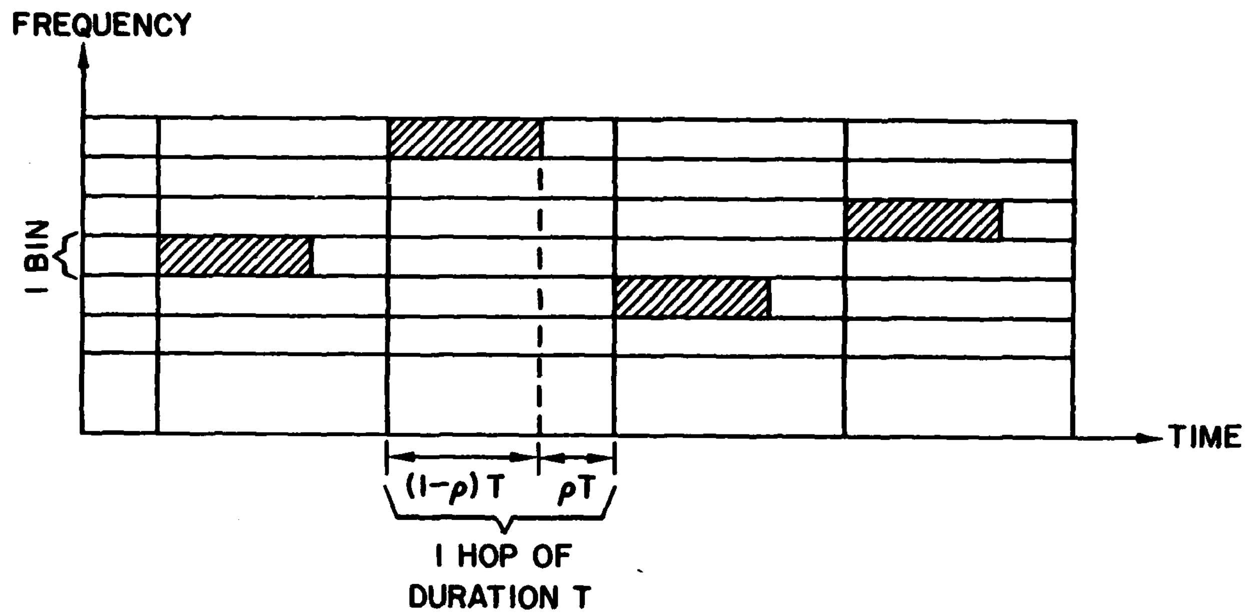

Fig. 4 Reduced Duty Cycle Frequency Hopping Signal.

regardless of degree of overlap, are assumed to necessitate symbol erasures.

In Appendix B we demonstrate that the symbol error (or erasure, if appropriate) probability for such a system is

$$P_k = 1 - (1 - 2(1-\rho)/q))^k, \qquad (9)$$

where $1-\rho$ is now the duty cycle. If it were possible to maintain q constant while ρ is increased, numerical results indicate that the performance of this system would be slightly better than that of our partial overlap model for M-ary FSK signaling for the same value of ρ. However, if we insist on a matched system (as indeed we should) then

$$\begin{aligned} q'(\rho) &= \text{number of frequency bins in matched reduced duty cycle system} \\ &= q(1-\rho). \qquad (10) \end{aligned}$$

Replacing q by $q'(\rho)$ in eq. (9) above results in

$$P_k = 1 - (1 - 2/q)^k, \qquad (11)$$

which is independent of ρ and is <u>identical</u> to the result presented in eq. (3) for a 100 percent duty cycle system in which overlaps cannot be tolerated, i.e., the "baseline system." The use of a reduced duty cycle in a matched system therefore has absolutely no effect on channel throughput.

A "Stretched Pulse" System with Fixed Bin Bandwidth

We can also consider the impact of modifying the baseline system that is initially incapable of tolerating overlaps and in which the dwell time is matched to the tone spacing. If one slows down the hopping rate and thus dwells longer at each hop it may be possible to tolerate partial overlaps.* In Figure 5 we illustrate such a system in which the hop duration, initially T_o, is lengthened to $T = T_o/(1-\rho)$ to permit a tolerable hop overlap of ρ. The data rate is thus reduced by a factor of $(1-\rho)$, and therefore the throughput measure must reflect this same factor of decrease. We assume that in this case the number of frequency bins remains constant at q; we do not increase it to maintain a matched system because the version of our model that we are now considering assumes that doing so would preclude the ability to tolerate partial overlap.

The performance of a Stretched Pulse System with Fixed Bin Bandwidth is discussed in Section 5 where we address the tradeoffs between slowing the signaling rate and the resulting achievable channel throughput. It does turn out that there is an optimum amount of "stretching" that yields maximum real throughput, higher than that of the baseline system, when the number of transmitting users is relatively large (e.g., greater than 29 for q = 100 frequency bins).

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* not just by means of a more sophisticated receiver, but by virtue of the longer dwell period itself.

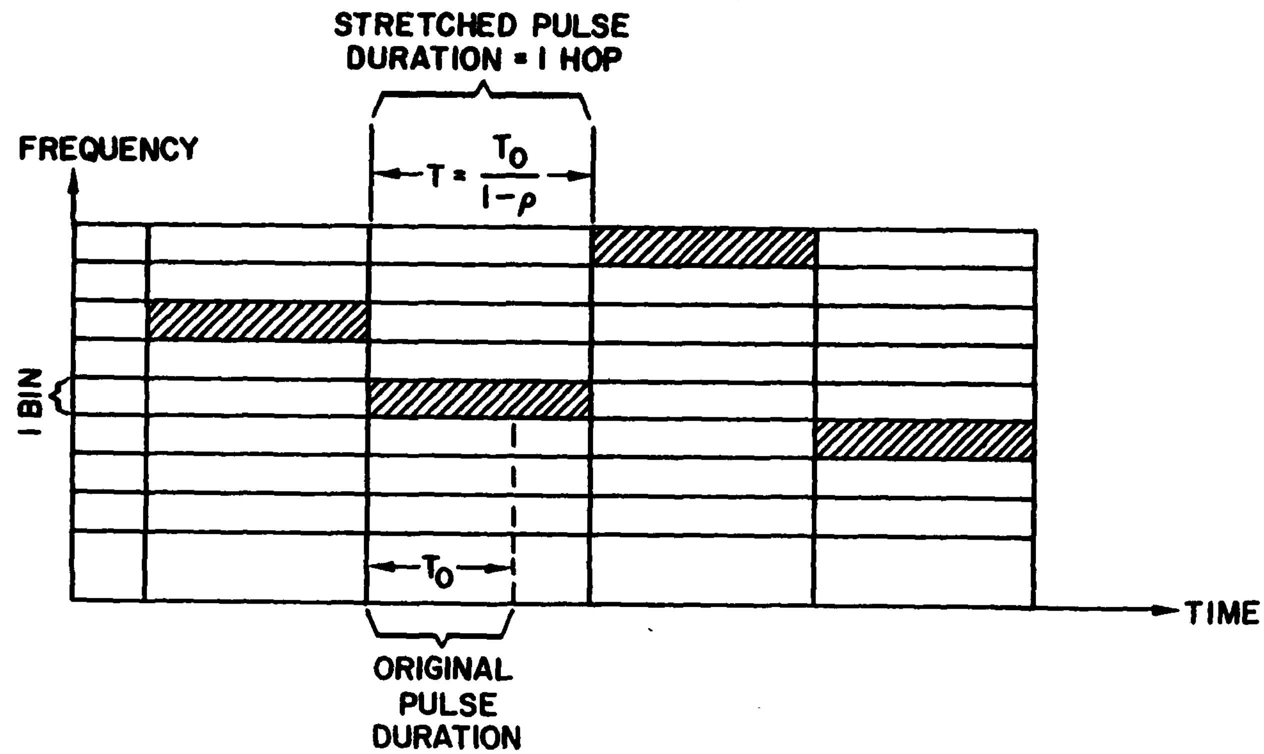

Fig. 5 "Stretched Pulse" Frequency Hopping Signal.

A "Stretched Pulse" System with Decreasing Bin Bandwidth

A further modification of the preceding system is one in which the lengthening of the dwell time is accompanied by a compressing of the frequency bins, to maintain the orthogonal tone spacing, if indeed this can be done while maintaining the overlap parameter at its value of ρ. This may be possible if the primary consideration in achieving nonzero ρ is related simply to the absolute duration of the dwell time rather than the dwell time's being greater than that corresponding to a matched system. If this can be done, then we would now have $q/(1-\rho)$ frequency bins. Performance would then be evaluated taking into account both the throughput reduction factor of $1-\rho$ (as a result of the reduced data rate) as well as the increased number of available frequency bins in the given bandwidth.

We will not continue comparing the differences between these system models. The purpose of this section has been to note that there are alternative physical reasons that can create systems that are capable of tolerating partial hop overlaps. The main emphasis of our paper is to show the means of analysis and the gains in performance for systems that do indeed tolerate such partial hop overlap.

5. PERFORMANCE EVALUATION

The performance measures we have considered are the probability of incorrect packet reception (evaluated using eq. (4) with the appropriate limits used in the summation), and the resulting channel throughput (obtained through the use of eq. (1). We first consider the pessimistic models for other-user interference in which all hits, regardless of the degree of overlap, result in loss of data.

Figure 6 illustrates the probability of incorrect packet reception (including detectable as well as undetectable codeword errors) as a function of the number of users that are transmitting simultaneously over a channel with 100 frequency bins. We have noted that virtually all packet errors are in fact detectable. Three curves are shown in this figure. The two upper curves were generated under the assumption that frequency hits are not detectable, and that they all result in symbol errors. The bottom curve was generated under the assumption that frequency hits are detectable, and that the corresponding symbols can be erased. We have referred to such a system as our "baseline system." The packet error probability using the RS-(255,127) code with detectable hits is extremely low, and falls below the range of the plots. The ability to detect frequency hits and erase the corresponding symbols therefore results in a considerable increase in the number of simultaneous users that the FH channel can support.

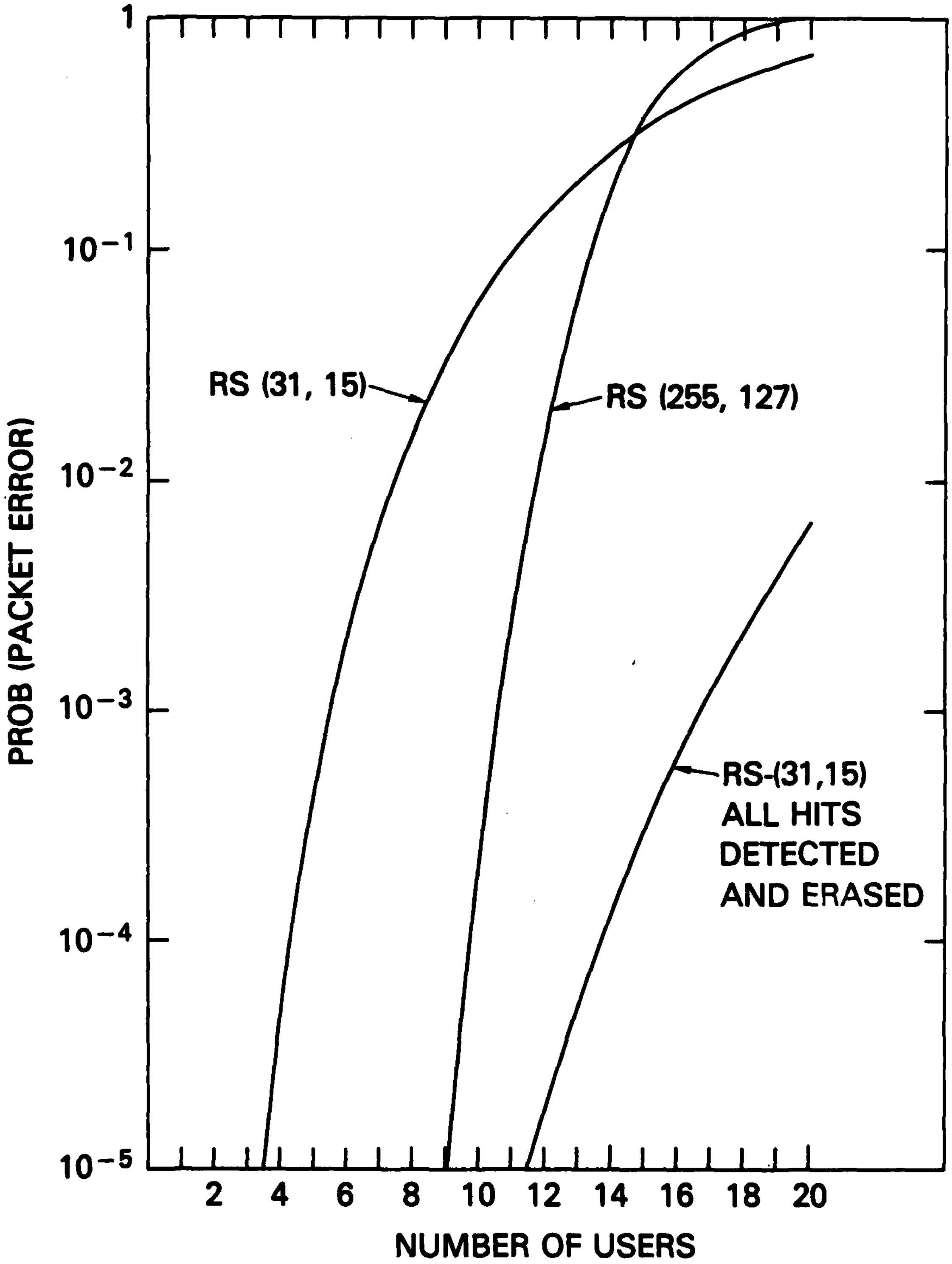

Fig. 6 Packet error probability for a noiseless asynchronous multiple user FH channel; rate 1/2 RS coding; 100 frequency bins.

Expected channel throughput for the RS codes discussed above is shown in Figure 7 for the noiseless channel, as well as for a channel in which the noise-induced symbol error probability (in the absence of other-user interference) is 0.1. Also shown in Figure 7 is the noiseless case in which hits are recognized and erased (our baseline system), resulting in substantial performance improvement. We have noted that the detection of hits would be straightforward in applications involving noiseless channels.

We now consider the model in which partial overlaps less than or equal to ρ can be tolerated. Figure 8 illustrates the packet erasure probability as a function of ρ for $q = 100$ frequency bins, $k+1 = 50$ simultaneous users each of which transmits continuously, and several alphabet sizes (M) where we are using RS-(M-1,(M-2)/2) codes, which have rate approximately equal to 1/2. Note that for ρ less than about 0.4 the shorter codes result in lower packet erasure probability. In most practical applications, however, where there are symbol errors resulting from channel noise, the use of longer codes is preferable because of their greatly improved ability to detect codeword errors. We note, however, that for the M-ary FSK signaling scheme considered here higher alphabet sizes are less bandwidth efficient than lower ones; for a fixed hopping rate the bandwidth is proportional to M, whereas the data rate is proportional to $\log_2 M$.

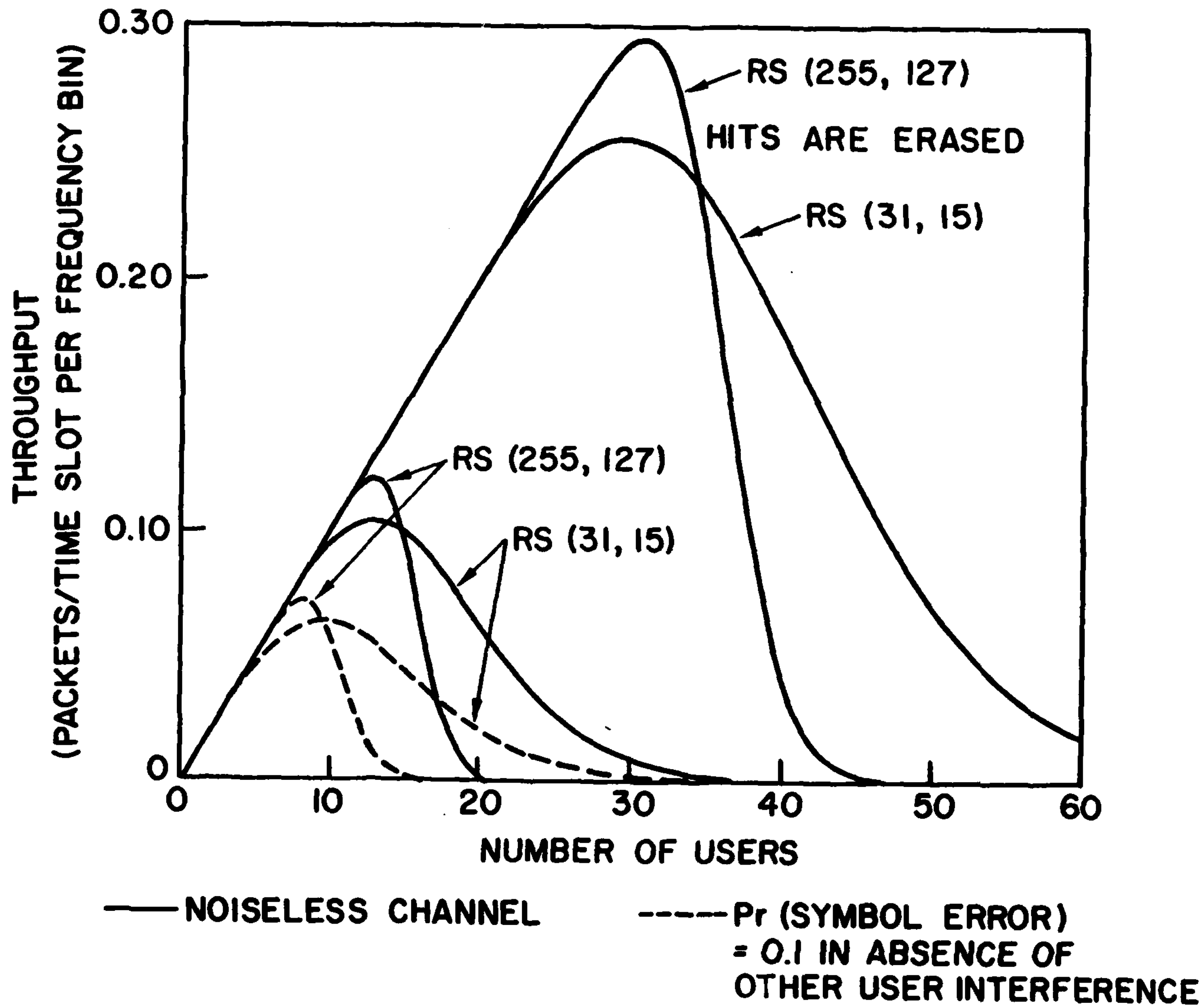

Fig. 7 Throughput per frequency bin of an asynchronous multiple user FH channel; rate 1/2 RS coding; 100 frequency bins.

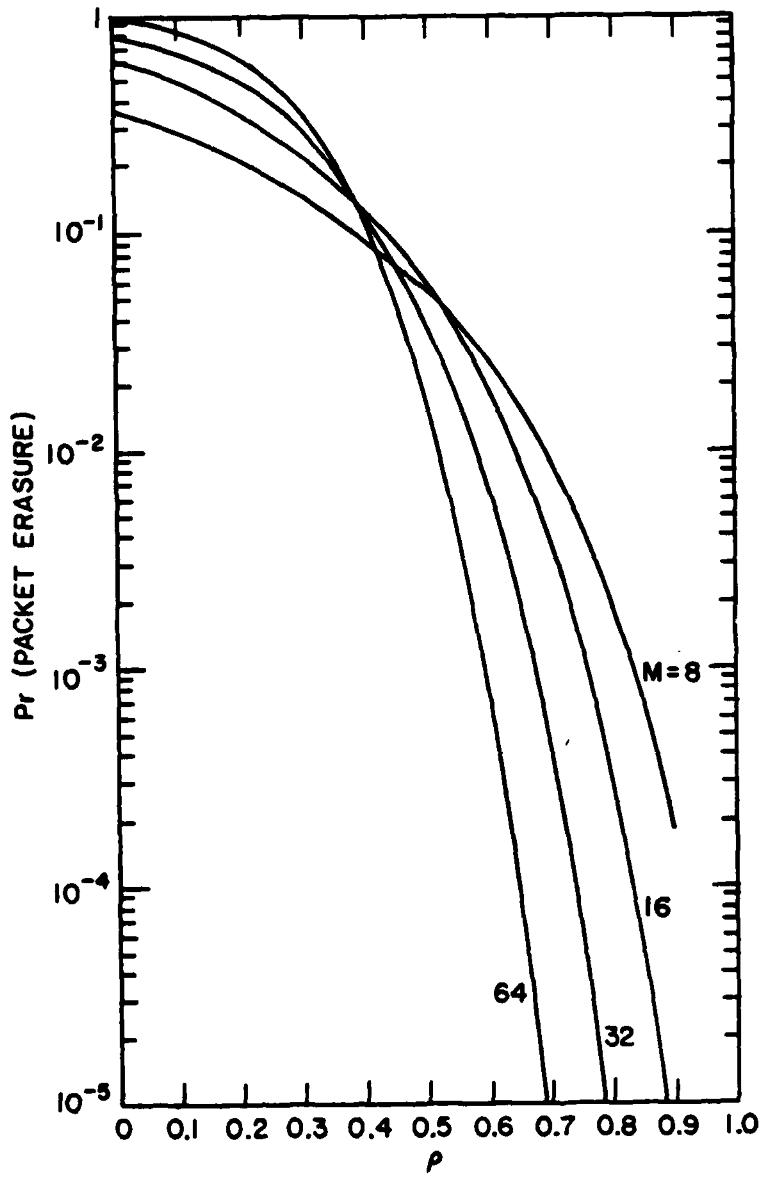

Fig. 8 Packet erasure probability for a noiseless asynchronous multiple user FH channel in which partial hits whose combined total overlap in any tone position is less than ρ can be ignored; M-ary FSK signaling; RS-(M-1,(M-2)/2) coding; 100 frequency bins; 50 users.

Figures 9 and 10 illustrate the packet erasure probability and throughput per frequency bin as a function of ρ for q = 100 frequency bins, M = 32, and RS-(31,15) coding for several values of k+1. The case of $\rho = 0$ corresponds to the curves representing detectable and erasable hits (our baseline system) shown in Figures 6 and 7. As ρ approaches 1 the total channel throughput (summed over all frequency bins) approaches the number of users; the throughput per frequency bin can actually be greater than one packet per time slot! It is difficult to estimate values of ρ that may be achievable in a practical system. Realistic values would depend on hopping rates and hardware implementation, as well as on the channel model, e.g., on noise levels in the case of noisy channels.

The relative importance of the two criteria, packet erasure probability and channel throughput, depends on the particular application. We have noted that even in a noisy environment in which hits cannot be recognized, virtually all packet errors are detectable even if they are not correctable. In cases where delays resulting from retransmission can be well tolerated then channel throughput would be more significant. In cases, however, where either retransmission delays cannot be tolerated or an acknowledgment mechanism cannot be implemented, then either packet error probability or bit error probability should be the performance measure. There is a tradeoff between packet error probability (or erasure probability if applicable) and throughput, which is most apparent through a comparison of

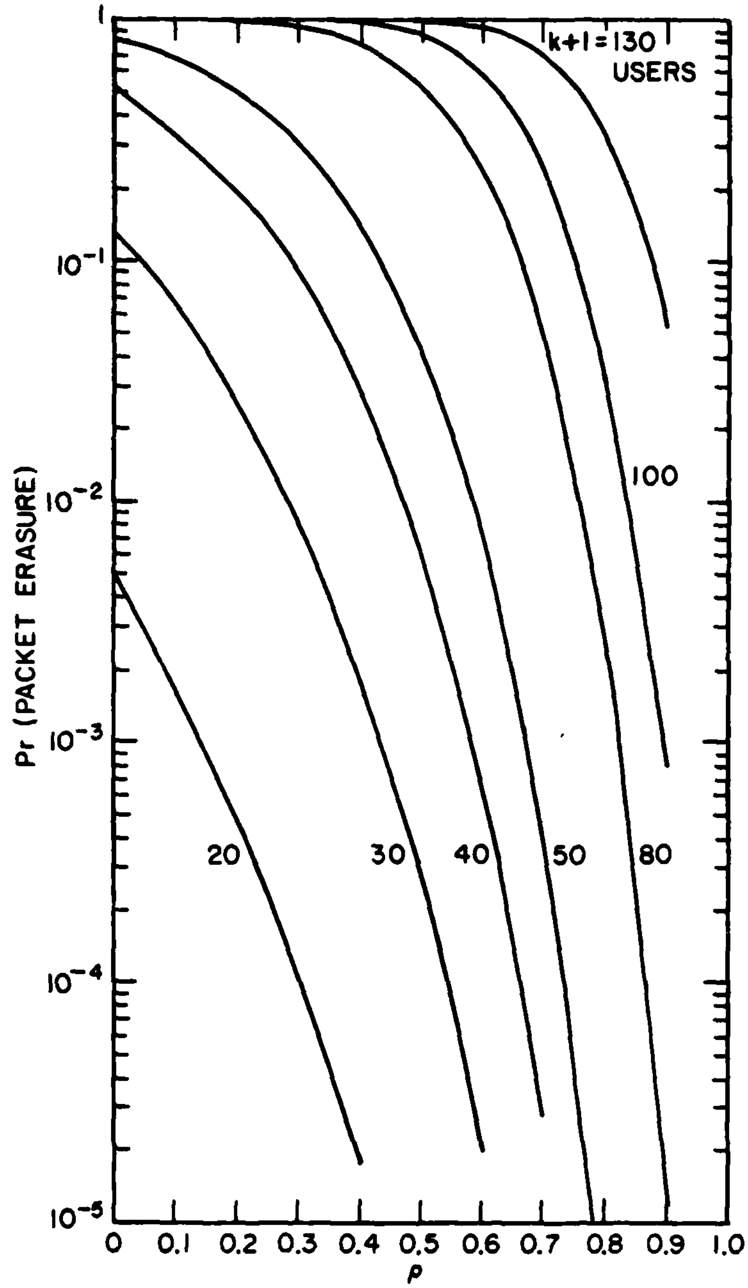

Fig. 9 Packet erasure probability for a noiseless asynchronous multiple user FH channel in which partial hits whose combined total overlap in any tone position is less than ρ can be ignored; 32-ary FSK signaling; RS-(31,15) coding; 100 frequency bins.

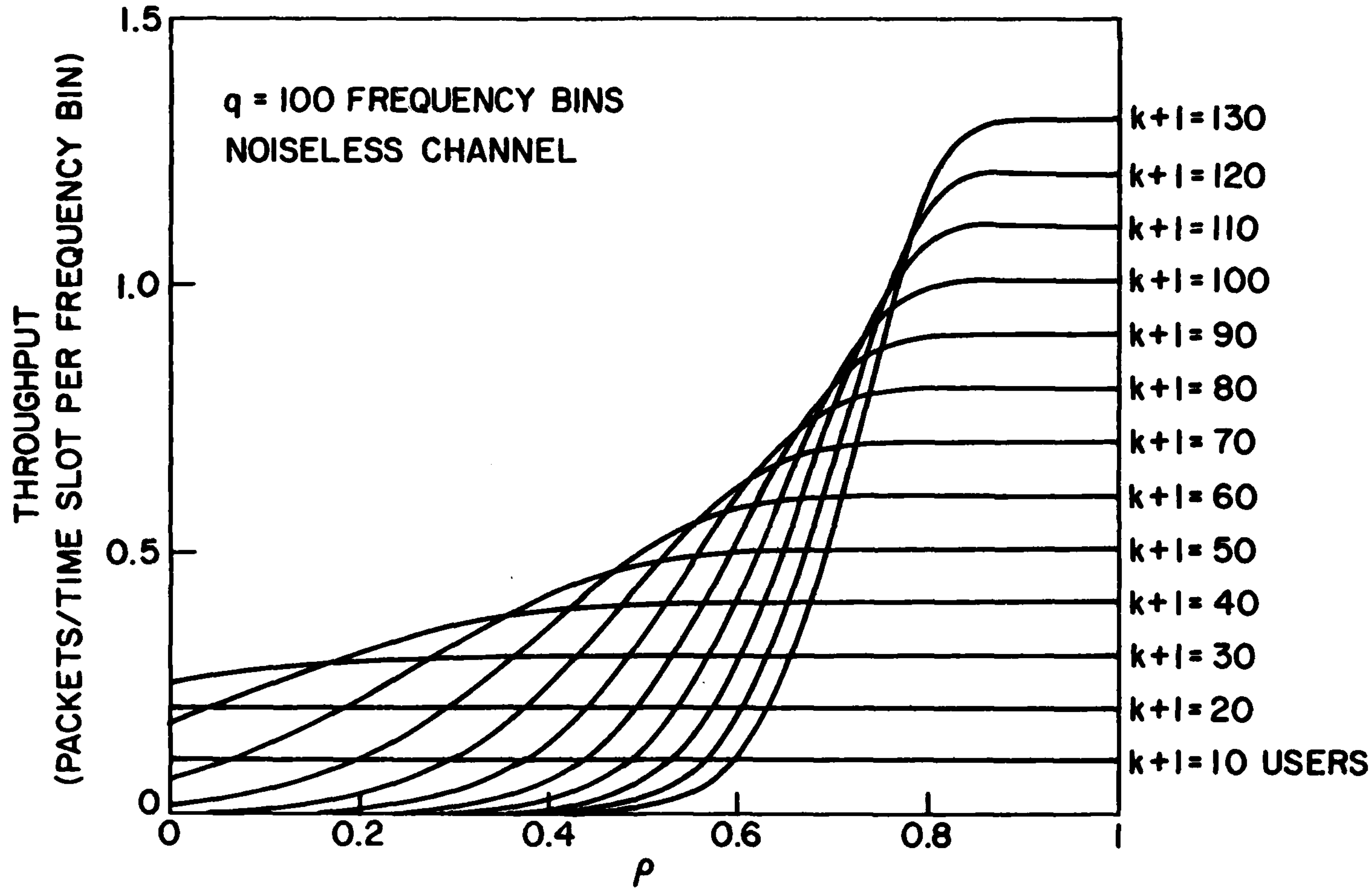

Fig. 10 Throughput per frequency bin of a noiseless asynchronous multiple user *FH* channel in which partial hits whose combined total overlap in any tone position is less than ρ can be ignored; 32-ary FSK signaling; RS-(31,15) coding; 100 frequency bins.

Figures 6 and 7. Although the probability of packet error increases as the number of simultaneous users increases, the total throughput (summed over all users) is maximized for a particular optimum number of users.

In Section 4 we considered several variations on the idea of tolerable partial overlap. One of these we termed the "Stretched Pulse System with Fixed Bin Bandwidth." The throughput performance of this system is obtained by multiplying the curves of Figure 10 by the factor $(1-\rho)$ to reflect the fact that the ability to tolerate partial overlap results from a pulse stretching that lowers the data rate. (In our basic model for systems that can tolerate partial overlap we assume that the ability to do so does not require a lengthening of the pulses beyond that of a matched system, i.e., one in which the tone spacing is equal to the reciprocal of the hop duration.) The resulting throughput is illustrated in Figure 11. Note that there is an optimum value of ρ that varies with the number of users. For small values of $k+1$ the optimum value of ρ is 0, indicating that we should not have slowed down our system. The maximum throughput that we can obtain in this system is 25.22 packets per time slot and frequency bin, which is achieved for $\rho = 0$ and a total of $k+1 = 29$ users.

As the number of users increases beyond 29 the optimum value of ρ increases and the maximum achievable throughput decreases slightly. For example, for $k+1 = 130$ a throughput of 23.36

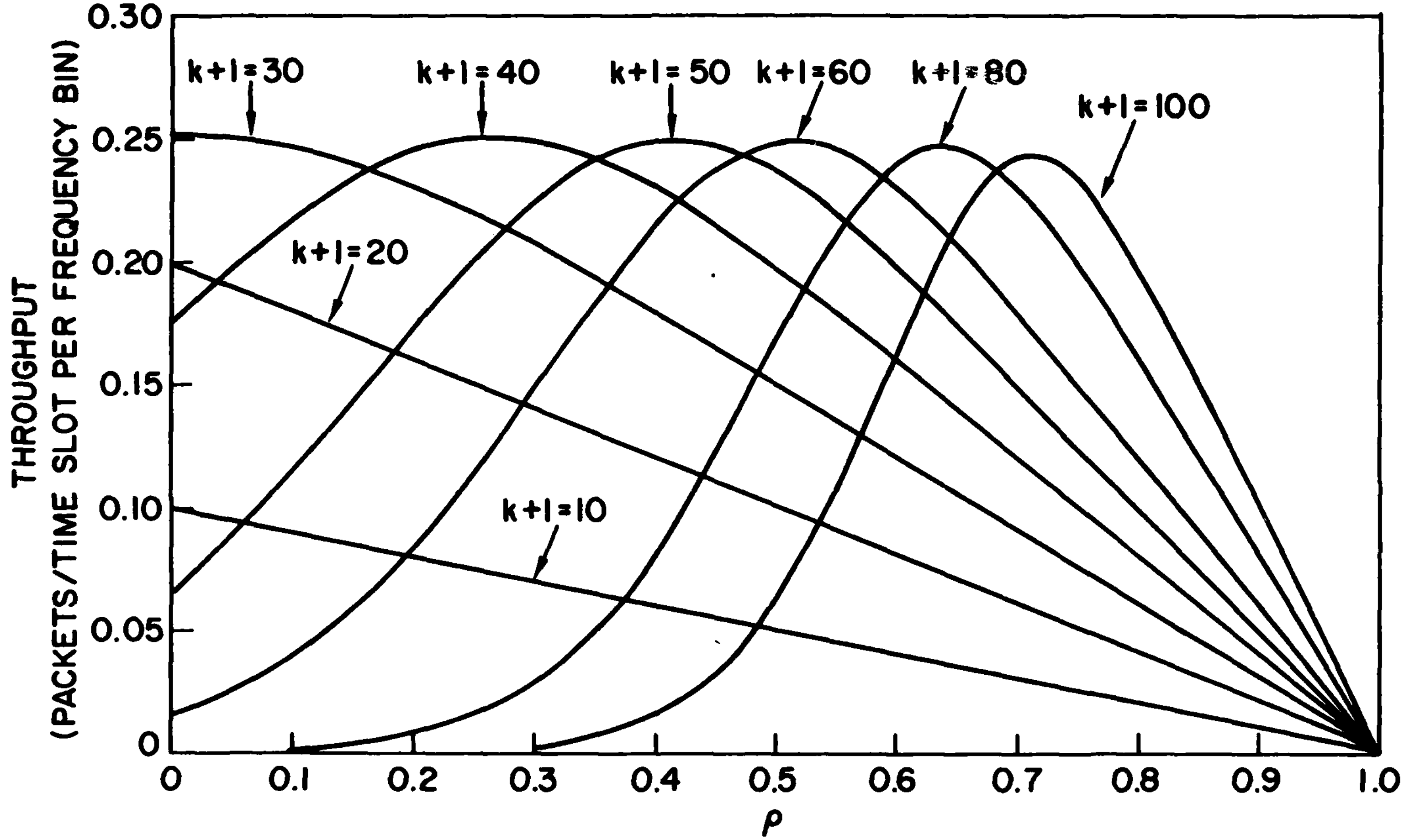

Fig. 11 Normalized throughput per frequency bin of a noiseless asynchronous multiple user FH Stretched Pulse System With Fixed Bin Bandwidth; partial hits whose combined total overlap in any tone position is less than ρ can be ignored; 32-ary FSK signaling; RS-(31,15) coding; 100 frequency bins.

results for $\rho = 0.775$. However, as k+1 increases the performance becomes increasingly sensitive to the value of ρ; thus, ρ can be chosen to produce a high value of throughput only if there is a good estimate of the number of active users.

The fact that the maximum throughput is achieved for $\rho = 0$ and $k+1 = 29$ suggests that limiting the number of active users may be preferable to pulse stretching. This would be especially true if one considered the probability of incorrect packet reception to be of importance in addition to throughput. The choice between the two alternative schemes of pulse stretching and reducing the number of users should be based on the characteristics of channel traffic and the ability to design "time-domain" channel access schemes to coordinate and control the users' demands on the channel in the specific environment one is operating in.

We should finally observe that the implementation of the proposed receiver (that is, one which can discriminate against partially overlapping interfering signals) is motivated further by the increased protection it can provide to a slow FH system against repeater jammers. Thus, in addition to improved throughput performance, increased antijamming capability can also be achieved.

6. COMPARISON OF PARALLEL BINARY FSK TO M-ARY FSK SIGNALING

We noted earlier that the ability to ignore interfering signals that are present for a sufficiently small fraction of the hop duration is based on the fact that in an M-ary FSK signaling format the M-ary signal is constant throughout the hop duration. Consider now the case of signaling with multiple parallel binary FSK tones, which is equivalent to the previous scheme in the following sense. Under the parallel scheme $\log_2 M$ binary FSK tones (requiring a total of $2\log_2 M$ tone positions*) are transmitted simultaneously in each hop, as shown in Figure 12, rather than a single M-ary FSK tone. We use the same coding scheme as in the M-ary case; i.e., a single M-ary Reed-Solomon symbol (which now consists of $2\log_2 M$ parallel tones) is transmitted in each hop. An advantage of using multiple binary tones is a reduced bandwith requirement (by a factor of $2\log_2 M/M$). A disadvantage, however, is a variable power level envelope, which is undesirable when amplifiers are operating in nonlinear regions, as is often the case. The relative performance of the two types of signaling schemes is dependent on the interference environment.

The analysis for the case of parallel binary tones follows directly from that for the M-ary signaling case. The basic difference is that two <u>different</u> signals (i.e., two different

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* There are $\log_2 M$ tones for "mark" and $\log_2 M$ tones for "space" in each bin.

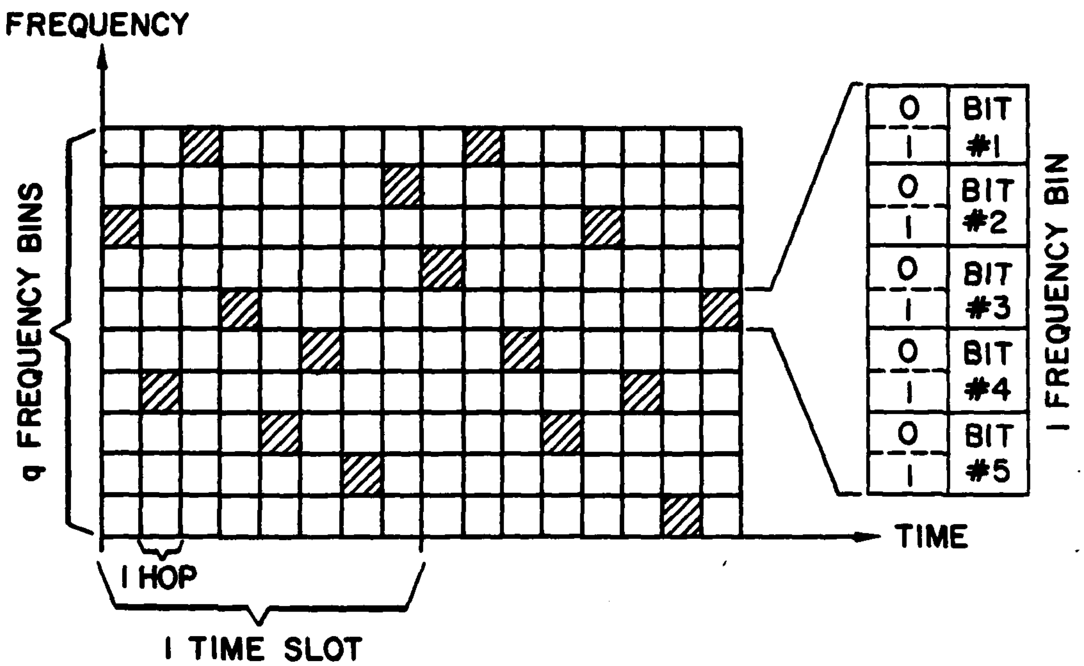

Fig. 12 Frequency hopping channel with $\log_2 M$ binary FSK symbols transmitted in parallel per hop and one packet (codeword) transmitted per time slot (example shown for M = 32).

$\log_2 M$-bit symbols), can produce a combined overlap greater than ρ in a tone position, because they may have one or more binary symbols in common. That is, the signals are no longer orthogonal as they were in the M-ary signaling case. We again use eq. (5) to define the total overlap τ; however, τ_L (τ_R) now refers to the maximum left (right) overlap of all the signals sharing the frequency bin, regardless of whether or not the interfering signals represent the same $\log_2 M$-bit symbol.*

The probability, then, of a symbol erasure, conditioned on the presence of m other users in the same frequency bin as the desired signal, is therefore easily obtained from eq. (A.19) by replacing j with m:

$$P(e|m) = 1 - (m+1)(\rho/2)^m. \tag{12}$$

In Appendix C we demonstrate that the symbol erasure probability, given that k other users are transmitting on the channel, is given by,

$$p_k = 1 - \left(\frac{\rho}{q} + 1 - \frac{2}{q}\right)^k - k\frac{\rho}{q}\left(\frac{\rho}{q} + 1 - \frac{2}{q}\right)^{k-1}. \tag{13}$$

As in the M-ary case, the packet erasure probability is evaluated using eq. (4) with a lower summation limit of $i = 2t+1$; the throughput is then obtained using eq. (2).

----

* Note that if there were a single "left" and single "right" interfering user, and if these signals were complements of each other (i.e., all mark and space symbols were reversed) then we would be able to distinguish between them, as in the M-ary signaling case. We do not, however, wish to complicate the analysis by considering this relatively infrequent, singular occurrence.

The throughput as a function of ρ, as the number of users is varied from 10 through 130, is shown in Figure 13 for the case of K = 5 parallel signals (and thus 10 parallel tone positions), q = 100 frequency bins, and RS-(31,15) coding. The performance curves are similar in appearance to, but not as good as, those for the M-ary signaling case, because in the parallel signaling case all "left" and "right" overlapping signals in the same frequency bin can combine with each other, regardless of whether or not they represent the same M-ary symbol. The ability to implement large values of ρ permits considerable improvement over the baseline system ($\rho = 0$), as was demonstrated earlier for the M-ary signaling case. Also note that, as shown in Figure 14, the normalized performance of the "stretched pulse" system shows behavior similar to that of the MFSK stretched pulse system, but slightly inferior to it.

A true comparison between the parallel binary and M-ary signaling schemes should reflect their relative bandwidth requirements by expressing throughput in terms of a common unit of bandwidth. If we normalize the measure of bandwidth to be that of a frequency bin in the M-ary signaling case then we must multiply the throughput curves presented in Figure 13 by the factor $M/2\log_2 M$, which is 3.2 for M = 32. Throughput per unit bandwidth is thus considerably better in the parallel binary case if we are operating with values of ρ and k+1 that produce

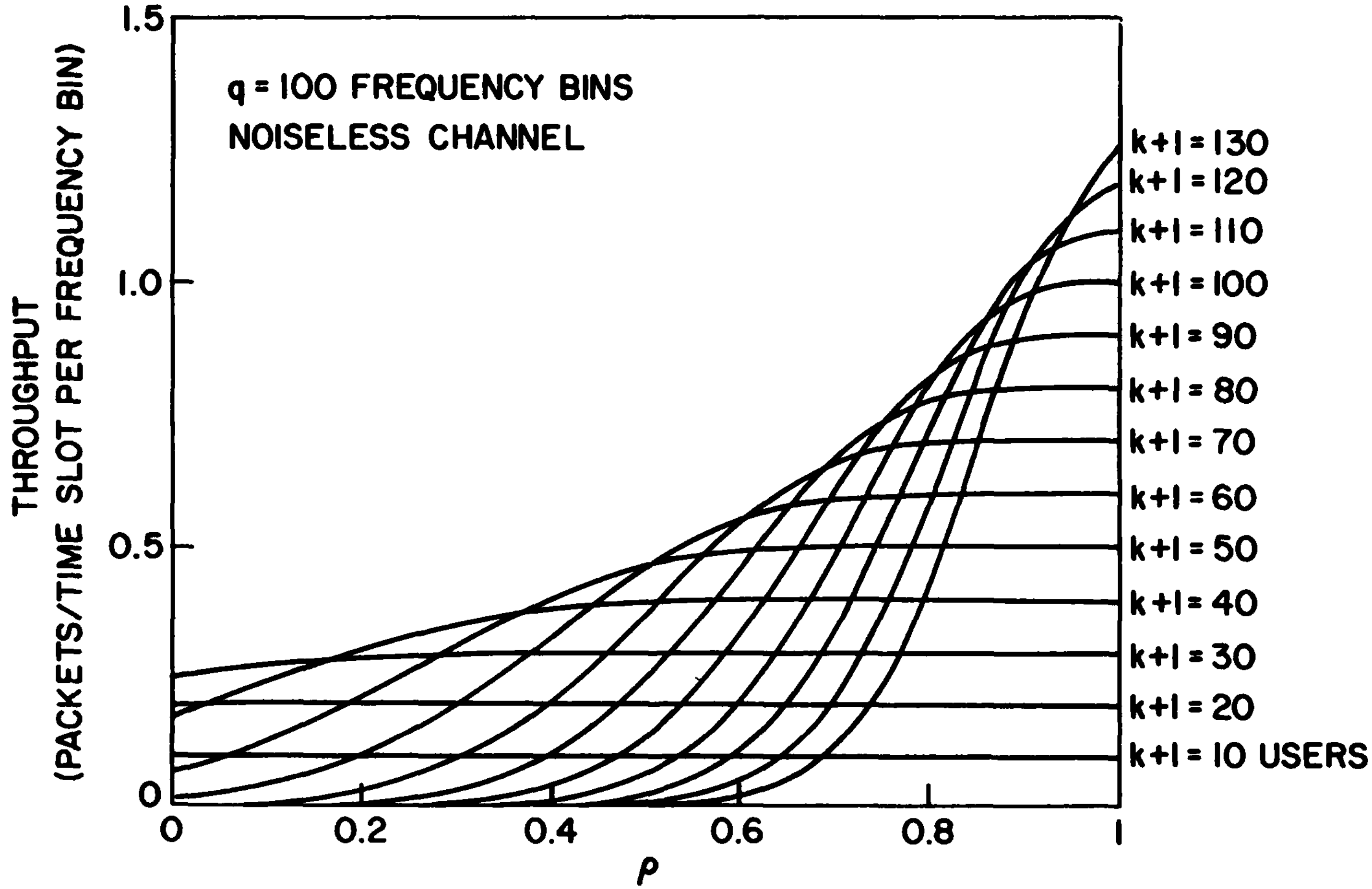

Fig. 13 Throughput per frequency bin of a noiseless asynchronous multiple user FH channel in which partial hits whose combined total overlap is less than ρ can be ignored; 5 binary FSK tones in parallel; RS-(31,15) coding; 100 frequency bins.

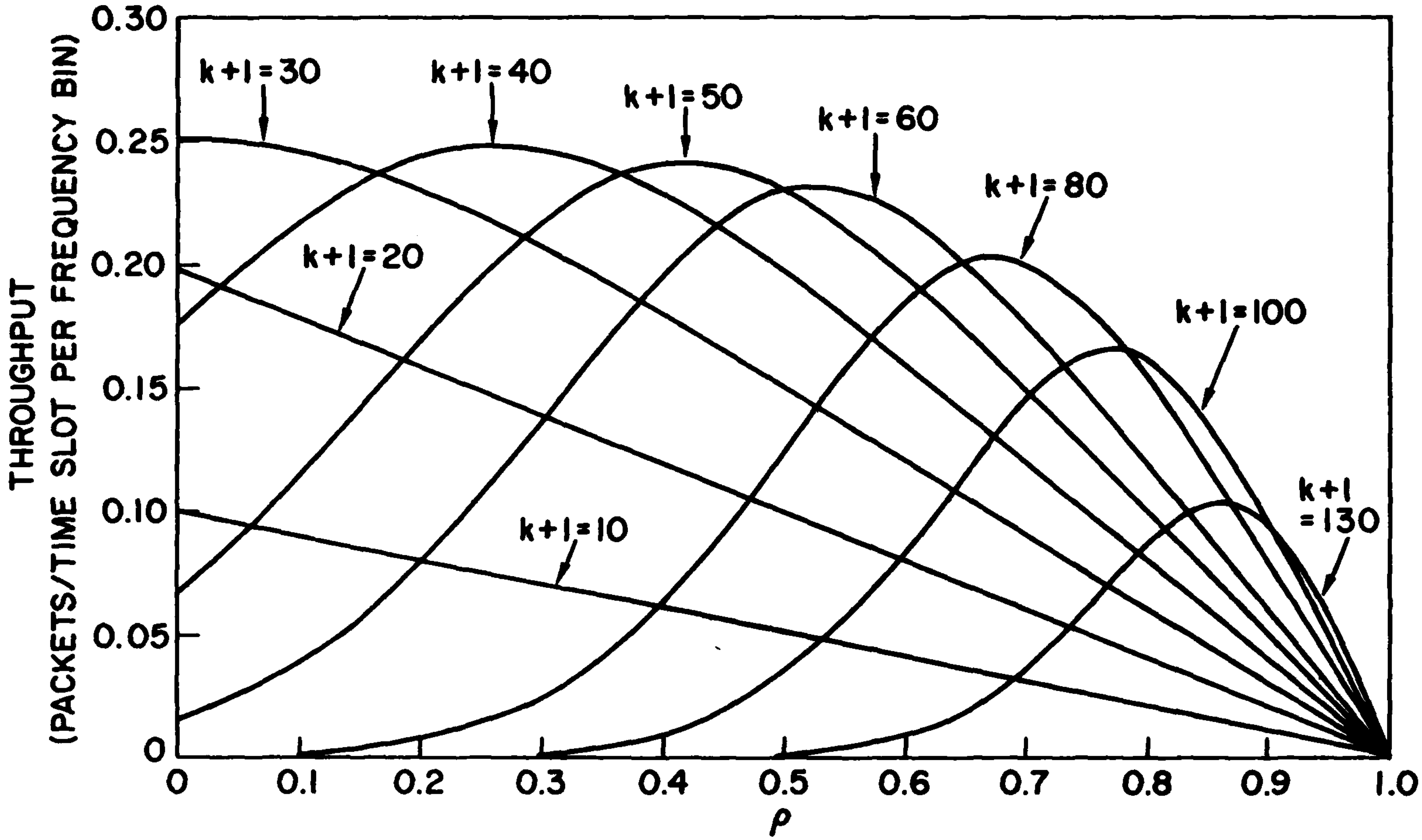

Fig. 14 Normalized throughput per frequency bin of a noiseless asynchronous multiple user FH Stretched Pulse System With Fixed Bin Bandwidth; partial hits whose combined total overlap is less than ρ can be ignored; 5 binary FSK tones in parallel; RS-(31,15) coding; 100 frequency bins.

throughput that is near the upper envelope of the curves. If the value of ρ is too low for a given number of users, then the M-ary signaling scheme is more bandwidth efficient.

7. CONCLUSIONS

The number of users that can share a wideband channel by means of Code Division Multiple Access (CDMA) techniques and the resulting performance depend on the modulation/coding scheme, channel characteristics, and receiver implementation. In this paper we have considered frequency hopping (FH) spread spectrum multiple access systems in which Reed-Solomon coding is used to correct burst errors caused by other-user interference in a packet-switched environment. Under the model considered each packet is encoded as a RS codeword, one symbol of which is transmitted in each hop. We have considered a noiseless channel model in which the only interference is that caused by other users. It will be interesting to extend the approach of this paper to the case of a noisy channel model.

We have proposed a model for other-user interference in which frequency hits can be detected. Furthermore, it is assumed that hops (symbols) can be correctly received, despite partial overlaps by other users' signals, provided that the overlap is a sufficiently small fraction of the hop duration. Such an interference model is valid provided that the signal remains constant throughout each hop duration and that a sufficiently sophisticated receiver is used. We have considered both M-ary FSK signaling and parallel binary tones. We have also considered various interpretations of the capability of tolerating partial overlap, and the implications on spectral efficiency.

For both the M-ary and parallel binary signaling cases we have derived exact expressions for the probability of successful symbol reception and the resulting probability of correct packet reception and channel throughput as a function of the number of channel users, number of frequency bins, alphabet size, and tolerable symbol overlap ρ. The ability to discriminate against interfering signals that are present for a sufficiently small fraction of the hop duration results in dramatic increases in throughput as ρ is increased from 0 to 1.

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APPENDIX A

EVALUATION OF THE CONDITIONAL SYMBOL ERASURE PROBABILITY

In this appendix we evaluate,

$$P(e|m) = \Pr[\text{symbol erasure} \mid \text{given } m \text{ other users in same frequency bin as desired signal}].$$

This quantity depends on the alphabet size M and the tolerable overlap parameter ρ.

It is the same as the conditional probability that the overlap in one or more tone positions is greater than ρ, given m. Note that when two or more overlapping signals share the same tone position we must consider their combined overlap τ, which was defined in Section 3. We must therefore characterize the distribution of the m other users' signals among the tone positions, and then for each such distribution of other-users' signals evaluate the probability that the overlap is indeed greater than ρ in one or more tone positions. To do so we define a "state" variable $\underline{n}_m$ to represent the tone position occupancy distribution within a frequency bin in which m other signals are present in addition to the desired one. We let

$$\underline{n}_m = (n_m(1), n_m(2), \ldots, n_m(m)), \tag{A.1}$$

where,

$n_m(j)$ = number of distinct tone positions in the bin, each containing exactly j other signals, or equivalently, number of "bunches" of exactly j users occupying distinct tone positions in the bin.

For example, consider the case of $m = 5$. The state $\underline{n}_5 = (3,1,0,0,0)$ corresponds to a realization in which one tone position is occupied by two other users and three tone positions are singly occupied by other users.

In general, the states $\underline{n}_m$ that are realizable must satisfy two physical constraints. First, since there are a total of m other signals we have, for any state $\underline{n}_m$,

$$\sum_{j=1}^{m} j n_m(j) = m. \tag{A.2}$$

Also, the total number of occupied tone positions, which we denote by $\hat{n}$, can not be greater than either the alphabet size (M) or the number of other users in the frequency bin (m), i.e.,

$$\hat{n} = \sum_{j=1}^{T} n_m(j) \le \min(m,M) \tag{A.3}$$

Thus, states that violate either of these constraints cannot occur, and need not be considered in the analysis.

The conditional symbol erasure probability, given m, can then be expressed as,

$$P(e|m) = 1 - \sum_{\underline{n}_m} \Pr(\text{symbol success}|\underline{n}_m) R(\underline{n}_m), \tag{A.4}$$

where,

$R(\underline{n}_m)$ = probability of state $\underline{n}_m$ occurring.

The evaluation of $P(e|m)$ therefore requires:

1) The enumeration of all possible states $\underline{n}_m$, to perform the indicated summation.

2) The evaluation of the conditional probability distribution $R(\underline{n}_m)$ for each state $\underline{n}_m$.

3) The evaluation of Pr(symbol success$|\underline{n}_m$) for each state $\underline{n}_m$.

We now proceed through these three steps.

1 -- Enumeration of the States

Given that there are M tone positions we must enumerate the states $\underline{n}_m$ that can occur for any particular value of m, i.e., all states for which eqs. (A.2) and (A.3) are satisfied. This problem is equivalent to the construction of Young's lattice [9]. We proceed iteratively as follows:

Assume we know all states for a given value of m. (For example, we may start with the trivial case of m = 1, for which the only state is $\underline{n}_1 = (1)$). For each state $\underline{n}_m$ that is consistent with the presence of m other users in the frequency bin of interest we determine the states $\underline{n}_{m+1}$ that can be generated as one additional user is added to the system. To do so we first consider the case in which the new (m+1)st user has chosen a tone position not previously chosen by any of the first m users. The number of singly occupied tone positions thus increases by 1 while the number of tone positions containing i other users (i = 2,3,...,m) remains unchanged. The new states are thus generated by the following procedure.

For each state $\underline{n}_m$, set:

$$n_{m+1}(1) = n_m(1) + 1,$$
$$n_{m+1}(i) = n_m(i), \qquad i = 2,3,\ldots,m \qquad \text{(A.5)}$$

We now consider the case in which the new (m+1)st user has chosen one of the tone positions already chosen by one of the first m users. If that tone position already contained i users, then it would now contain i+1 users, thus decrementing the number containing i users by 1 while incrementing the number containing i+1 by 1. The new states are thus generated by the following procedure.

For each state $\underline{n}_m$ and every $n_m(i) \neq 0$, set:

$$
\begin{aligned}
n_{m+1}(i) &= n_m(i) - 1 \\
n_{m+1}(i+1) &= n_m(i+1) + 1.
\end{aligned}
\tag{A.6}
$$

Note that duplicate identical states are generated by this process, because two different $\underline{n}_m$ states can evolve to the same $\underline{n}_{m+1}$ successor state. Such duplicate states are easily recognized and thinned out in this iterative procedure. Also with this procedure, states for which the number of occupied tone positions ($\hat{n}$) is greater than the alphabet size M may be generated. Such states are discarded.

As an example we show in Table 1 the resulting states for $m \leq 5$ (and $M \geq m$).

Table 1 -- Enumeration of states for $m \leq 5$ and $M \geq m$.

m = 1: (1)

m = 2: (2,0), (0,1)

m = 3: (3,0,0), (1,1,0), (0,0,1)

m = 4: (4,0,0,0), (2,1,0,0), (0,2,0,0), (1,0,1,0), (0,0,0,1)

m = 5: (5,0,0,0,0), (3,1,0,0,0), (1,2,0,0,0), (2,0,1,0,0)
(0,1,1,0,0), (1,0,0,1,0), (0,0,0,0,1)

2 -- The State Probability Distribution: $R(\underline{n}_m)$

We now determine the probability distribution $R(\underline{n}_m)$. We first observe that we are not interested in which of the M tone positions in a frequency bin are occupied by a specific number of signals, but only in the number of such tone positions. Thus, each state $\underline{n}_m$ corresponds to several different realizations, all of which are equally likely.

Our approach is to consider m users transmitting in the same frequency bin as the desired signal, each of which places a signal into one of the M tone positions; each tone position is chosen equally likely with probability 1/M. We now consider the sequence in which some subset of the M tone positions is filled as we examine the m users, which are numbered from 1 to m. We want to realize the state $\underline{n}_m = (n_m(1), n_m(2), \ldots n_m(m))$.

There are numerous ways to do so. Let us consider a realization in which the first $n_m(1)$ users all choose different tone positions; these are followed by $n_m(2)$ pairs of users, such that the two members of each pair choose the same tone position as each other, but different from all those previously chosen; these are followed by $n_m(3)$ groups of three users, such that the three members of each triplet choose the same tone position as each other, but different from all those previously chosen, etc. We note that $n_m(m)$, and in fact $n_m(j)$ for $j > m/2$, can not be greater than 1.

As we consider the users numbered from 1 to m we evaluate the probability that the conditions corresponding to the specific chosen realization of $\underline{n}_m$, which we denote $\underline{I}_m$, are satisfied. We note that the probability of any other specific realization corresponding to the same state $\underline{n}_m$ is identical to that which we derive here.*

We first consider the $n_m(1)$ singly occupied tone positions. The first user chooses a tone position at random; thus, the probability that he picks a previously unchosen tone position is simply $M/M = 1$. The second user also chooses a tone position at random; the probability that he picks a previously unchosen tone position is $(M-1)/M$. Continuing in the same manner, user $n_m(1)$ chooses a tone position that was not previously chosen with probability $[M-(n_m(1)-1)]/M$. Thus,

Pr(first $n_m(1)$ users choose different tone positions)

$$= \frac{M}{M} \frac{(M-1)}{M} \frac{(M-2)}{M} \cdots \frac{(M-(n_m(1)-1))}{M} \qquad \text{(A.7)}$$

Given that the first $n_m(1)$ users have chosen different tone positions, we now evaluate the probability that we then have $n_m(2)$ pairs of users that choose the same tone position as each other, but different from previously chosen tone positions. Thus, user $n_m(1)+1$ must choose a tone position that is different from that of the first $n_m(1)$ signals, an event that occurs with probability $[M-n_m(1)]/M$. User $n_m(1)+2$ must choose the <u>same</u> tone position as user $n_m(1)+1$, an event that occurs with probability

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* This can be easily seen by a renumbering of the users.

1/M. Similarly, user $n_m(1)+3$ must choose a tone position not previously chosen (resulting in a probability of $[M-n_m(1)-1]/M$), and user $n_m(1)+4$ must choose the same tone position as user $n_m(1)+3$ (resulting in 1/M).

Continuing in the same manner, we finally obtain

$$\Pr(\underline{I}_m) = \underbrace{\left(\frac{M}{M}\right)\left(\frac{M-1}{M}\right)\left(\frac{M-2}{M}\right) \cdots \left(\frac{[M-(n_m(1)-1)]}{M}\right)}_{n_m(1) \text{ factors}}$$

$$\cdot \underbrace{\left(\frac{M-n_m(1)}{M^2}\right)\left(\frac{M-n_m(1)-1}{M^2}\right) \cdots \left(\frac{M-n_m(1)-[n_m(2)-1]}{M^2}\right)}_{n_m(2) \text{ factors}}$$

$$\cdot \underbrace{\left(\frac{M-n_m(1)-n_m(2)}{M^3}\right) \cdots \left(\frac{M-n_m(1)-n_m(2)-[n_m(3)-1]}{M^3}\right)}_{n_m(3) \text{ factors}}$$

$$\vdots$$

$$\cdot \underbrace{\frac{M-n_m(1)-n_m(2)-\ldots-[n_m(m)-1]}{M^m}}_{n_m(m) \text{ factors } (\leq 1)}$$

$$= \frac{M(M-1)(M-2)\ldots[M-(n_m(1)+n_m(2)+n_m(m)-1)]}{M^{n_m(1)}\,(M^2)^{n_m(2)} \cdots (M^m)^{n_m(m)}}$$

$$= \frac{M!}{M^m(M-\hat{n})!} \tag{A.8}$$

where $\hat{n}$ is the total number of occupied tone positions. We note that several of the $n_m(i)$'s may be equal to zero without affecting the derivation.

Thus, we can write,

$$R(\underline{n}_m) = N(\underline{n}_m)Pr(\underline{I}_m) \tag{A.9}$$

where,

a) $Pr(\underline{I}_m)$ = the conditional probability of one particular realization of state $\underline{n}_m$.

and,

b) $N(\underline{n}_m)$ = the number of equally likely realizations in the class defined by $\underline{n}_m$.

We now determine $N(\underline{n}_m)$, the number of equally likely realizations of $\underline{n}_m$. This determination is identical to that of finding the number of different partitions of a set of m objects into classes of $n_m(j)$ groups, each group having j objects, for j = 1,2,...,m. From [9] we obtain,

$$N(\underline{n}_m) = \frac{m!}{\prod_{j=1}^{m} (j!)^{n_m(j)} \; n_m(j)!} \tag{A.10}$$

Combining this result with the probability of a specific realization results in

$$R(\underline{n}_m) = \frac{M!\ m!}{M^m (M-\hat{n})! \prod_{j=1}^{m} (j!)^{n_m(j)} \ n_m(j)!} \qquad \text{(A.11)}$$

As an example, in Table A.1 we evaluate eqs. (A.3, A.8, A.10, and A.11) for all states corresponding to m = 3.

Table A.1 -- Evaluation of state probabilities for m = 3.

$\underline{n}_3$	$\hat{n}$	$Pr(\underline{I}_3)$	$N(\underline{n}_3)$	$R(\underline{n}_3)$
(3,0,0)	3	$\frac{(M-1)(M-2)}{M^2}$	1	$\frac{(M-1)(M-2)}{M^2}$
(1,1,0)	2	$\frac{(M-1)}{M^2}$	3	$\frac{3(M-1)}{M^2}$
(0,0,1)	1	$\frac{1}{M^2}$	1	$\frac{1}{M^2}$

3 -- The Evaluation of Symbol Success Probability

We now evaluate the probability of successful symbol reception for each state $\underline{n}_m$. To do so requires two steps:

a) Evaluate $Pr(\tau<\rho|j)$, the probability that the total overlap τ is less than ρ in a particular tone position, given that j users transmit in that tone position.

b) Evaluate the resulting conditional symbol success probability, given state $\underline{n}_m$.

a) Evaluation of $Pr(\tau<\rho|j)$

We need the probability distribution of the total overlap caused by j other users in a single tone position. Let us fix a particular tone position. In general, there will be α other users overlapping at the left (leading) edge and $\beta = j-\alpha$ other users overlapping at the right (trailing) edge of the symbol, as shown earlier in Figure 3. Note that it is equally likely for an overlapping signal to be at the leading or trailing end of the hop.

We define,

$X(a)$ = time overlap with the symbol of interest of the a-th interfering symbol from the left, $a = 1,\ldots,\alpha$

$Y(b)$ = time overlap with the symbol of interest of the b-th interfering symbol from the right, $b = 1,\ldots,\beta$

The $X(a)$'s and $Y(b)$'s are independent and uniformly distributed over the interval (0,1). We also define,

$$\tau_L = \max_a (X(a))$$

$$\tau_R = \max_b (Y(b)),$$

The combined total overlap of all users in the particular tone position we are considering is therefore,

$$\tau = \min(1, \tau_L + \tau_R). \quad \text{(A.12)}$$

Thus we need,

$$\Pr(\tau < \rho \mid j) = \int_0^\rho f_\tau(z|j)\,dz, \quad 0 < \rho < 1, \quad \text{(A.13)}$$

where $f_\tau(z|j)$ is the conditional probability density function of τ given j. There is a discontinuity in $f_\tau(z|j)$ at the value of z equal to 1 because, as defined in eq. (A.12), $\tau = 1$ whenever $\tau_L + \tau_R \geq 1$; physically, this means that the "left" and "right" overlapping signals not only overlap with the desired signal, but they actually overlap with each other as well. A value of $\tau = 1-\varepsilon$ (for some small positive ε) means that, for the tone position we are examining, there is some clear interval within the hop duration in which there is no overlap with an other user signal. A value of ρ greater than or equal to 1, on the other hand, is uninteresting in that it would imply that all interference can be discriminated against with probability 1. Therefore, we only consider the case of ρ strictly less than 1.

As a result of the independence of τ_L and τ_R, the density of τ is simply the convolution of their densities. As it can easily be shown, the distribution for the maximum of n independent random variables uniform in (0,1) is given by:

$$f(x) = nx^{n-1}, \qquad 0 < x < 1. \tag{A.14}$$

We therefore obtain,

$$\begin{aligned} f_\tau(z) &= \int_0^z \beta x^{\beta-1} \alpha (z-x)^{\alpha-1} dx \\ &= \alpha \int_0^z (z-x)^{\alpha-1} dx^\beta \\ &= \alpha x^\beta (z-x)^{\alpha-1} \Big|_0^z + \alpha \int_0^z x^\beta (\alpha-1)\ (z-x)^{\alpha-2} dx \end{aligned} \tag{A.15}$$

Repeated integration by parts yields ultimately,

$$f_\tau(z) = \frac{\alpha + \beta}{\binom{\alpha + \beta}{\beta}} z^{\alpha+\beta-1} \qquad 0 < z < 1. \tag{A.16}$$

Therefore, for $\rho < 1$, we have

$$\begin{aligned} \Pr(\tau < \rho) &= \int_0^\rho f_\tau(z) dz \\ &= \frac{\alpha + \beta}{\binom{\alpha + \beta}{\beta}} \int_0^\rho z^{\alpha+\beta-1}\, dz \\ &= \frac{\rho^{\alpha+\beta}}{\binom{\alpha + \beta}{\beta}} \end{aligned} \tag{A.17}$$

Hence,

Pr($\tau<\rho$ | α "left" users and j-α "right" users

in this tone position)

$$= \frac{\rho^j}{\binom{j}{\alpha}} \tag{A.18}$$

The j users in the same tone position are equally likely to interfere from the "left" or from the "right." Thus,

$Pr(\tau<\rho \mid j)$

= Pr($\tau<\rho$ | j others in same tone position)

$$= \sum_{\alpha=0}^{j} \frac{1}{\binom{j}{\alpha}} \rho^j \binom{j}{\alpha} \left(\frac{1}{2}\right)^{\alpha} \left(\frac{1}{2}\right)^{j-\alpha}$$

$$= \sum_{\alpha=0}^{j} \left(\frac{\rho}{2}\right)^j$$

$$= (j+1) \left(\frac{\rho}{2}\right)^j \tag{A.19}$$

b) Evaluation of $Pr(\text{symbol success} | \underline{n}_m)$

We now evaluate the probability of successful symbol reception given the state $\underline{n}_m$. We define,

$$T_j = Pr(\text{in each of the } n_m(j) \text{ tone positions, each containing exactly } j \text{ other users, we have total overlap with symbol of interest} < \rho)$$

$$= [Pr(\tau<\rho | j)]^{n_m(j)}$$

$$= [(j+1)(\tfrac{\rho}{2})^j]^{n_m(j)} \qquad \text{(A.20)}$$

where we have made use of the independence among the total overlap variables in different tone positions, given the number of users in each tone position. Making further use of this independence, we write,

$$Pr(\text{symbol success} | \underline{n}_m) = \prod_{j=1}^{m} T_j$$

$$= \prod_{j=1}^{m} [(j+1)(\tfrac{\rho}{2})^j]^{n_m(j)}$$

$$= \rho^m \prod_{j=1}^{m} [(j+1)(\tfrac{1}{2})^j]^{n_m(j)} . \qquad \text{(A.21)}$$

Note that this derivation is valid only for ρ strictly less than 1, as discussed earlier.

Remark

By using eq. (A.4) we can evaluate exactly $P(e|m)$ for arbitrary m. For example, the resulting expressions for $m \leq 5$ are as follows:

$$\begin{aligned}
P(e|1) &= 1 - \rho \\
P(e|2) &= 1 - \frac{\rho^2}{M}(M - 1.25) \\
P(e|3) &= 1 - \frac{\rho^3}{M^2}(M^2 - 0.75M + 0.25) \qquad \text{(A.22)} \\
P(e|4) &= 1 - \frac{\rho^4}{M^3}(M^3 - 1.5M^2 + 1.1875M - 0.375) \\
P(e|5) &= 1 - \frac{\rho^5}{M^4}(M^4 - 2.5M^3 + 3.4375M^2 - 2.5M + 0.75)
\end{aligned}$$

Similar closed form exact expressions can be obtained for any larger values of m. In our computations we have made the approximation that $P(e|m) = 1$ for $m > 5$, resulting in a slightly pessimistic performance evaluation. Although this approximation is not strictly valid, especially for relatively large values of ρ, it has little effect on the evaluation of packet erasure probability because m is rarely as large as 6, unless the number of users is equal to a significant fraction of the number of frequency bins. That this is true is observed from eq. (8) from which the values displayed in Table A.2 are obtained.

Table A.2 -- $Pr(m \geq 6)$ for $q = 100$ and several values of k.

k	$Pr(m \geq 6)$
30	.000025
40	.000137
50	.00048
60	.00127
70	.0028
80	.0055
90	.0096
100	.0155

APPENDIX B

EVALUATION OF SYMBOL ERASURE PROBABILITY FOR THE REDUCED DUTY CYCLE SYSTEM

In the reduced duty cycle system each hop interval now consists of a pulse of duration $(1-\rho)T$ followed by a silent interval of duration ρT, as shown in Figure 4.* We set $T = 1$ without loss of generality. As in Section 2 we consider a single hop of the desired signal. We first evaluate the probability that a hop of an arbitrary other user's signal overlaps with this hop of the desired signal. Note that in this system we are assuming that any amount of overlap results in destructive interference, either in the form of a symbol error or an erasure. We distinguish two cases.

<u>Case 1</u>: $\rho > 1/2$

In Figure B.1 we illustrate the relative timing between one hop of the desired signal and the two potentially interfering hops of an arbitrary other user. We denote by τ the relative time delay of the other user hop that begins during the time interval defined by the desired signal's hop of interest. Note that $0 \leq \tau < 1$.

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* We may also consider a system in which the pulse location is randomly located within the hop interval. In that case a similar calculation can be carried out, but an additional averaging over the pulse location will be required.

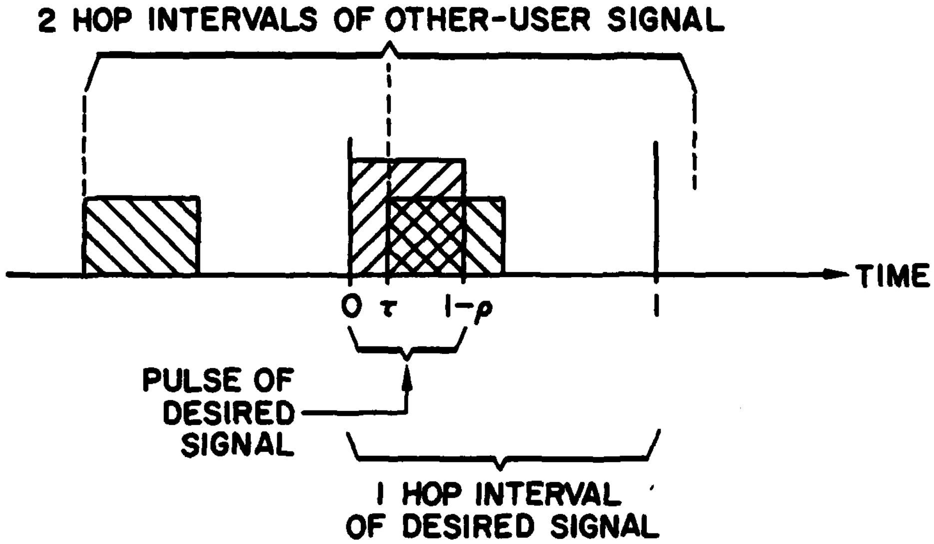

Fig. B.1 Relative timing between other-user signal and desired signal in Reduced Duty Cycle System, $\rho > 1/2$.*

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* Note that either (but not both) of the depicted hops of the other-user signal can be in the same bin as (and therefore interfere with) the depicted hop of the desired signal.

We distinguish three regions of values of τ:

a) $0 \leq \tau < 1-\rho$: the other use can only overlap from the right

b) $1-\rho \leq \tau < \rho$: the other user cannot overlap at all

c) $\rho \leq \tau < 1$: the other user can only overlap from the left

Since τ is uniformly distributed over the interval from 0 to 1 the *probability* that τ is in one particular interval is simply equal to the length of that interval. Also, the probability that one particular other user is in the same frequency bin as the desired signal is 1/q. The probability that either of two consecutive hops of that other user's signal causes interference is obtained by conditioning on the three possible regions of values of τ as defined above. Thus we obtain,

Pr(a particular other user causes interference)

$$= (1-\rho)(1/q) + 0 + (1-\rho)(1/q) = 2(1-\rho)/q.$$

<u>Case 2</u>: $\rho < 1/2$

We now consider $\rho < 1/2$, as shown in Figure B.2. The three regions of interest are now:

a) $0 \leq \tau < \rho$: the other user can only overlap from the right

b) $\rho \leq \tau < 1-\rho$: the other user may overlap either from the left or from the right*

c) $1-\rho \leq \tau < 1$: the other user can only overlap from the left

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* but not both because consecutive hops are assumed to be in different frequency bins; the probability that one of these two hops is in the same bin as the desired signal is 2/q.

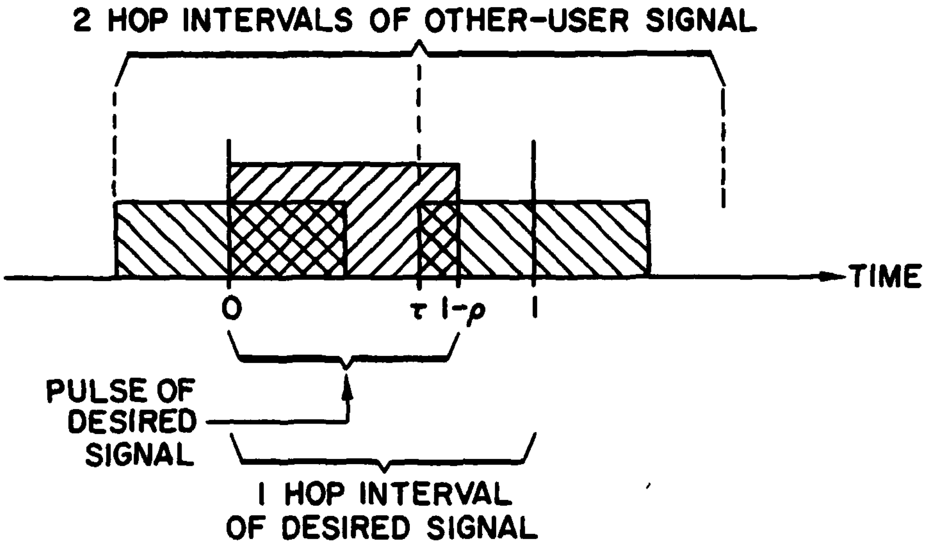

Fig. B.2 Relative timing between other-user signal and desired signal in Reduced Duty Cycle System, $\rho < 1/2$.

We therefore obtain,

Pr(a particular other user causes interference)

$$= \rho(1/q) + (1-2\rho)2/q + \rho(1/q) = 2(1-\rho)/q,$$

which is identical to the expression for the case of $\rho > 1/2$.

If there are k other users we follow the reasoning that led to eq. (3) and obtain,

$$P_k = 1 - (1-2(1-\rho)/q)^k.$$

APPENDIX C

EVALUATION OF SYMBOL ERASURE PROBABILITY FOR THE PARALLEL BINARY FSK CASE

The symbol erasure probability, given that there are k other channel users, can be expressed as (see eq. (7)),

$$p_k = \sum_{m=1}^{k} P(e|m)Q(m|k), \tag{C.1}$$

where $P(e|m)$ and $Q(m|k)$ are given by eqs. (12) and (8), respectively. We therefore obtain,

$$p_k = \sum_{m=1}^{k} [1-(m+1)(\tfrac{\rho}{2})^m][\binom{k}{m}(\tfrac{2}{q})^m(1-\tfrac{2}{q})^{k-m}]$$

$$= \sum_{m=1}^{k} \binom{k}{m}(\tfrac{2}{q})^m(1-\tfrac{2}{q})^{k-m} - \sum_{m=1}^{k}(m+1)(\tfrac{\rho}{2})^m\binom{k}{m}(\tfrac{2}{q})^m(1-\tfrac{2}{q})^{k-m} \tag{C.2}$$

which we express as,

$$p_k = \Sigma_1 - \Sigma_2 \tag{C.3}$$

The first summation, Σ_1, is easily recognized as the sum of a probability mass function, less the term corresponding to $m = 0$. We therefore obtain,

$$\Sigma_1 = 1 - (1 - 2/q)^k. \tag{C.4}$$

The second summation is,

$$\Sigma_2 = \sum_{m=1}^{k} m\left(\frac{\rho}{q}\right)^m \binom{k}{m}\left(1 - \frac{2}{q}\right)^{k-m}$$

$$+ \sum_{m=1}^{k} \binom{k}{m}\left(\frac{\rho}{q}\right)^m \left(1 - \frac{2}{q}\right)^{k-m} \qquad \text{(C.5)}$$

$$= A + B$$

To evaluate A we make use of the relationship

$$m\binom{k}{m} = \frac{m \; k!}{m!\,(k-m)!} = \frac{k!}{(m-1)!\,(k-m)!} = \frac{k(k-1)!}{(m-1)!\,(k-1-(m-1))!} \qquad \text{(C.6)}$$

Setting $n = m - 1$, we obtain

$$A = \sum_{n=0}^{k-1} \frac{k(k-1)!}{n!\,(k-1-n)!} \left(\frac{\rho}{q}\right)^{n+1} \left(1 - \frac{2}{q}\right)^{k-n-1}$$

$$= k\,\frac{\rho}{q} \sum_{n=0}^{k-1} \binom{k-1}{n} \left(\frac{\rho}{q}\right)^n \left(1 - \frac{2}{q}\right)^{(k-1)-n}$$

$$= k\,\frac{\rho}{q}\left(\frac{\rho}{q} + 1 - \frac{2}{q}\right)^{k-1}. \qquad \text{(C.7)}$$

To evaluate B we note that it is a binomial summation without the term corresponding to m = 0. We therefore obtain,

$$B = \sum_{m=1}^{k} \binom{k}{m} \left(\frac{\rho}{q}\right)^{m} \left(1 - \frac{2}{q}\right)^{k-m}$$

$$= \left(\frac{\rho}{q} + 1 - \frac{2}{q}\right)^{k} - \left(1 - \frac{2}{q}\right)^{k} \qquad (C.8)$$

Combining these results, we obtain finally,

$$p_k = 1 - \left(\frac{\rho}{q} + 1 - \frac{2}{q}\right)^{k} - k\frac{\rho}{q}\left(\frac{\rho}{q} + 1 - \frac{2}{q}\right)^{k-1}. \qquad (C.9)$$

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